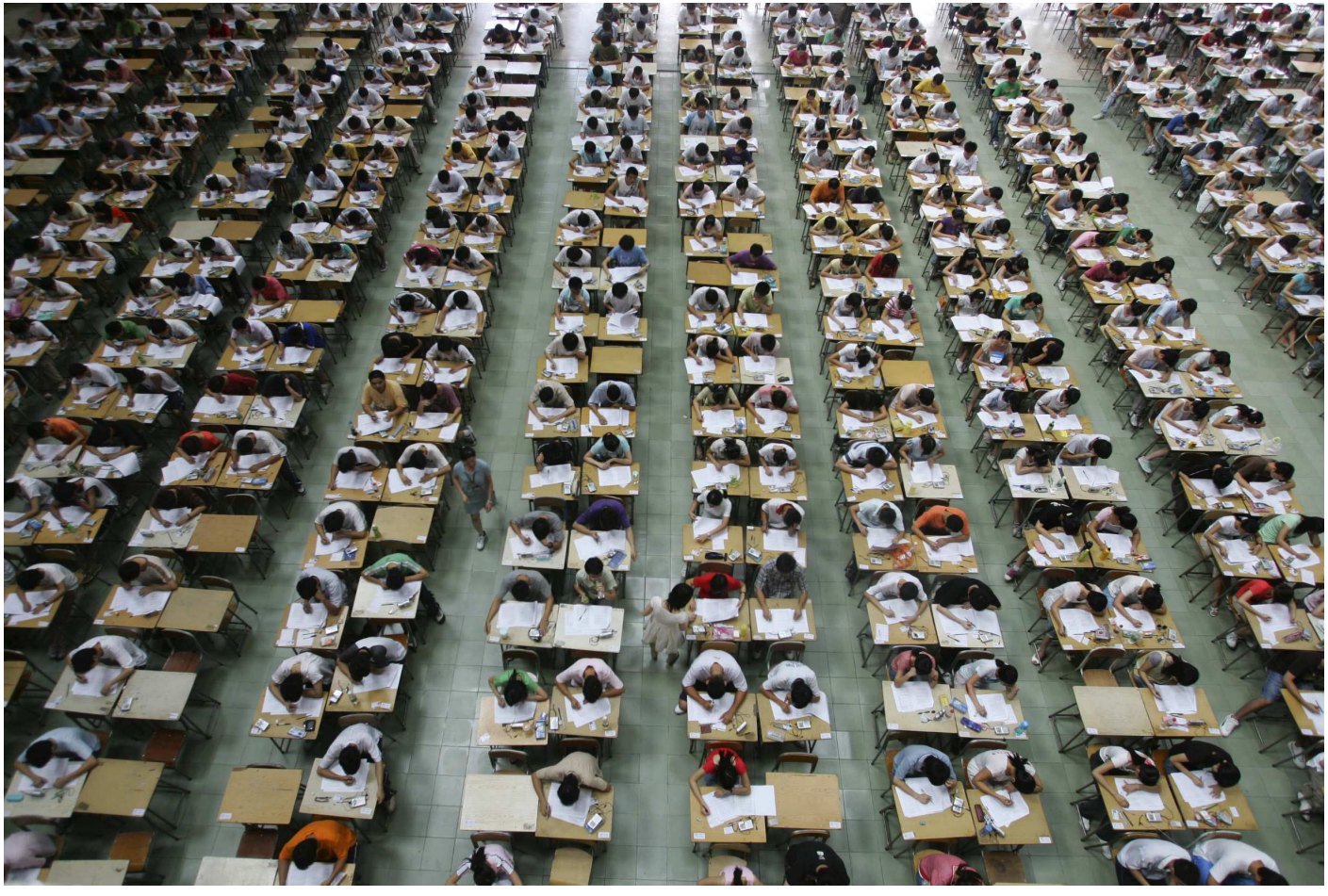
Geometry and Topology, or How Different a Mug and a Doughnut Really Are

Victoria LEBED, Research Fellow in Maths

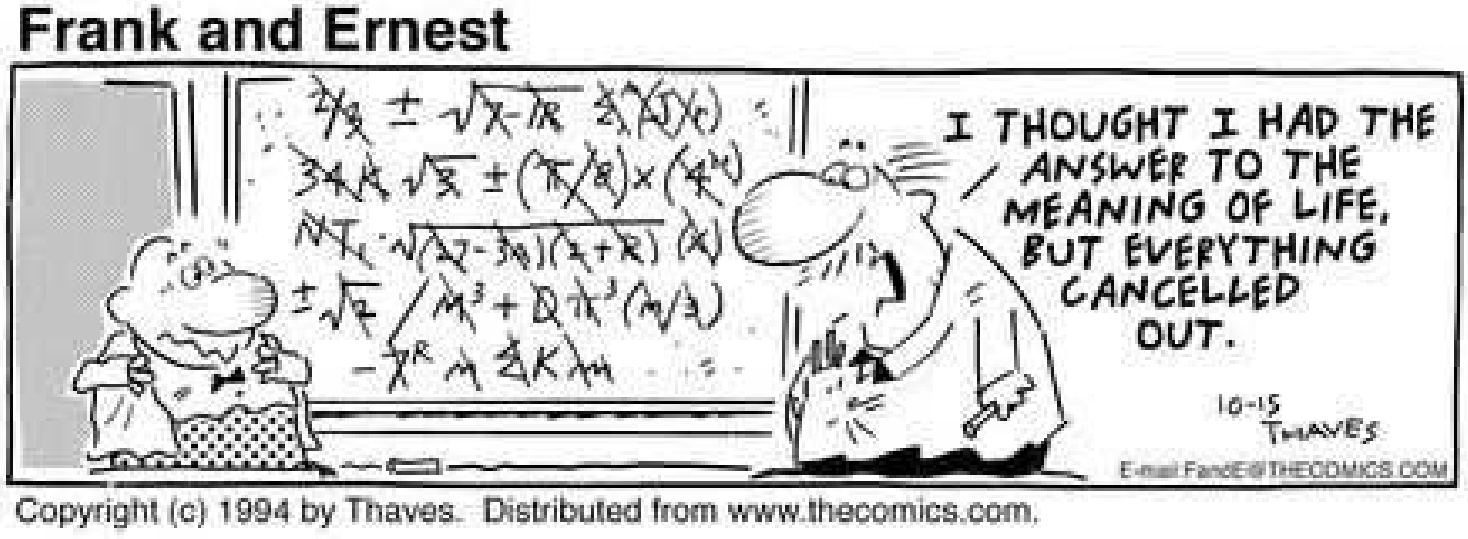
1 What is mathematics?

Is it this?



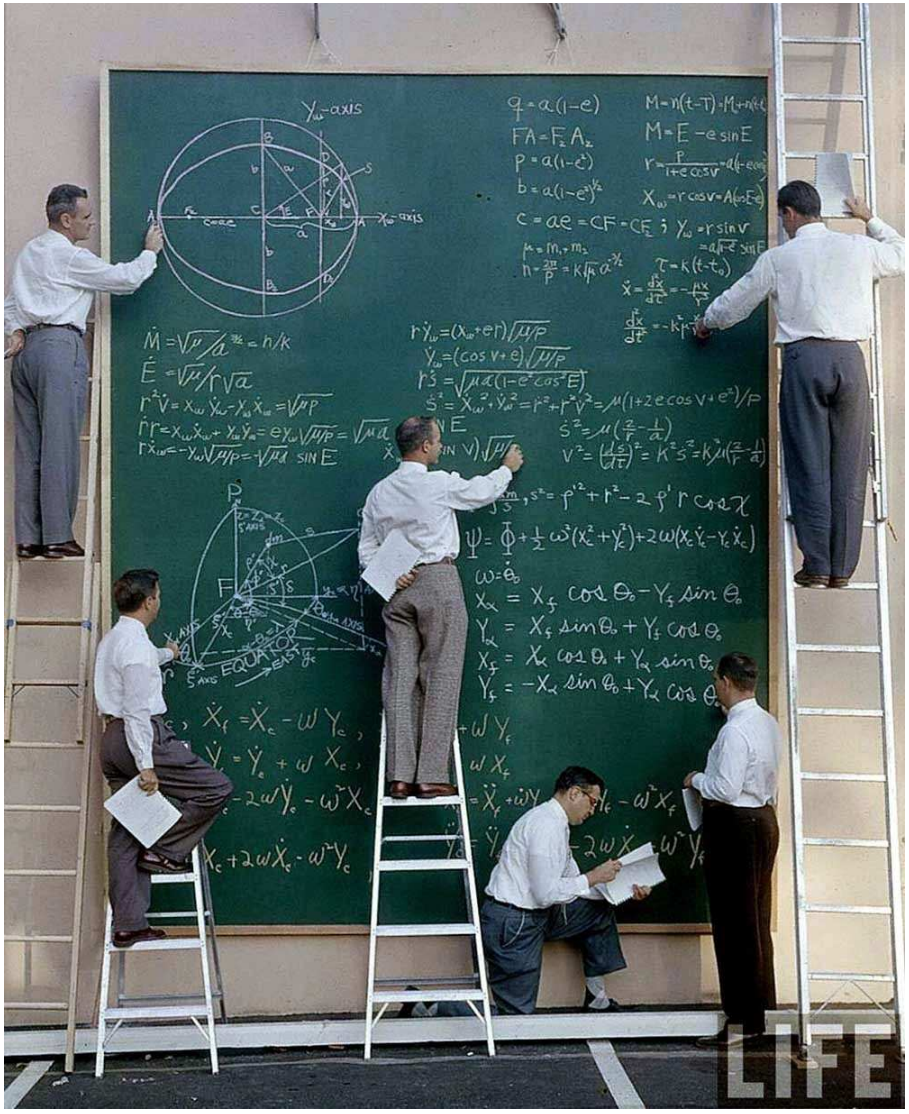
1 What is mathematics?

And this?



1 What is mathematics?

Sometimes yes:



NASA scientists with their board of calculations, 1961, *Life magazine*.

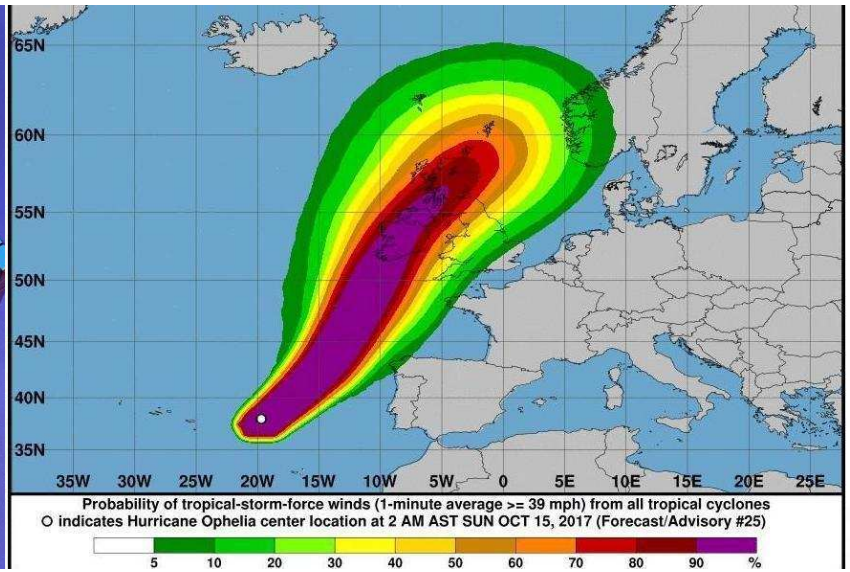
1 What is mathematics?

Sometimes yes:

A screenshot from *Hidden Figures*.

1 What is mathematics?

But maths is also all of this:



Maths is about understanding **mechanisms** and seeing **patterns**, not only about computing! It involves **creativity**, and is sometimes closer to arts and philosophy than to sciences.

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Fortune 500's Most Valued Characteristics in an Employee:

Characteristics	1999	1970
Teamwork	1	10
Problem Solving	2	12
Interpersonal Skills	3	13
Oral Communication	4	4
Listening Skills	5	5
Personal Career Development	6	6
Creative Thinking	7	7
Leadership	8	8
Goal Setting/Motivation	9	9
Writing	10	1
Organizational Effectiveness	11	11
Computational Skills	12	2
Reading Skills	13	3

2 Mugs and doughnuts

Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.

Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.

2 Mugs and doughnuts

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- ✓ Examples of applications:

1) surveying;
2) construction;
3) astronomy;
4) the shape of the universe.

Topology

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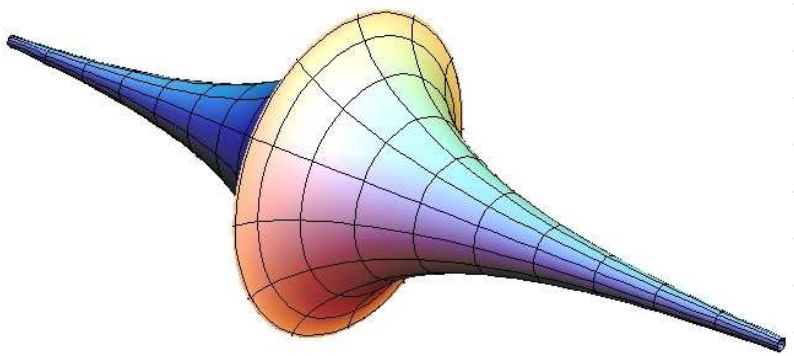

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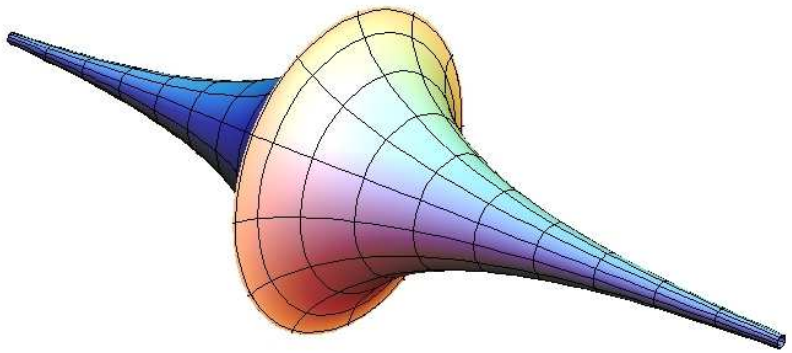

Topology

✓ Overall shape, deformations.
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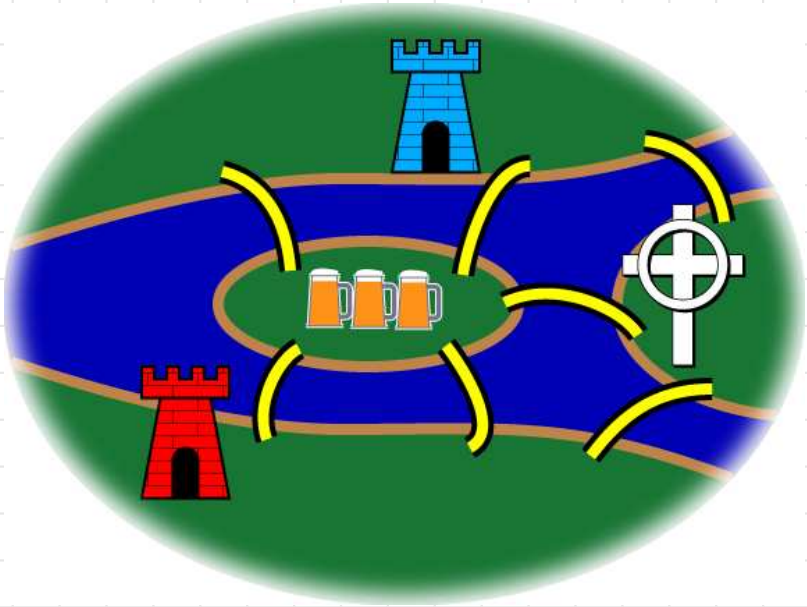
✓ Examples of applications:
1) Seven Bridges of Königsberg Problem;



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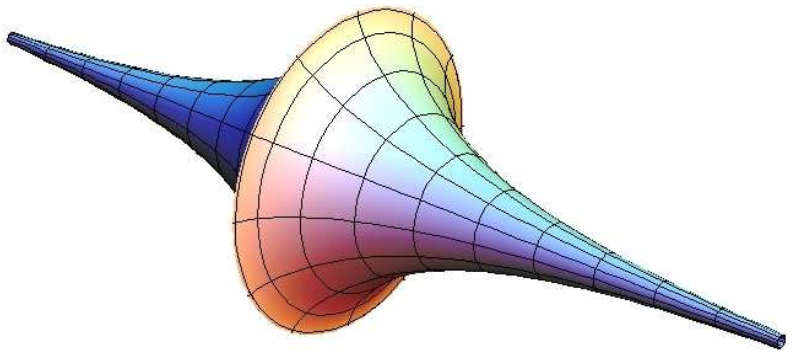
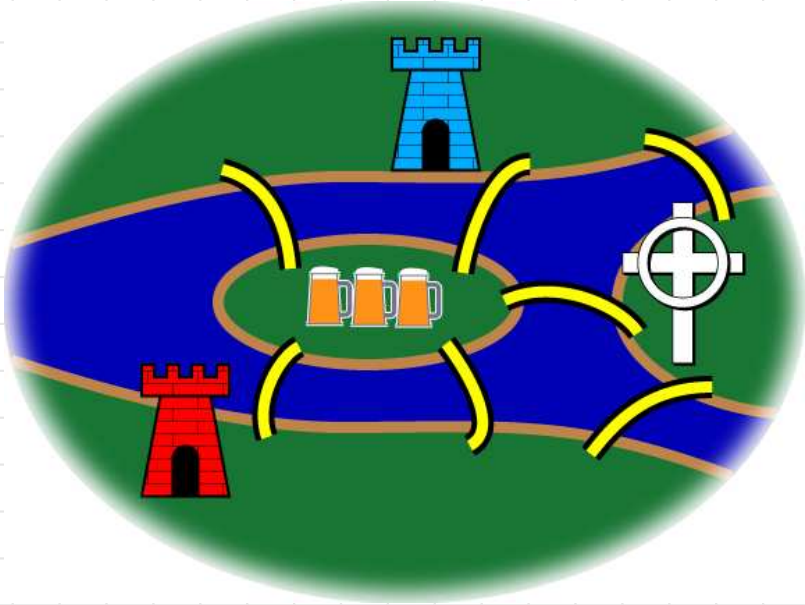


Topology

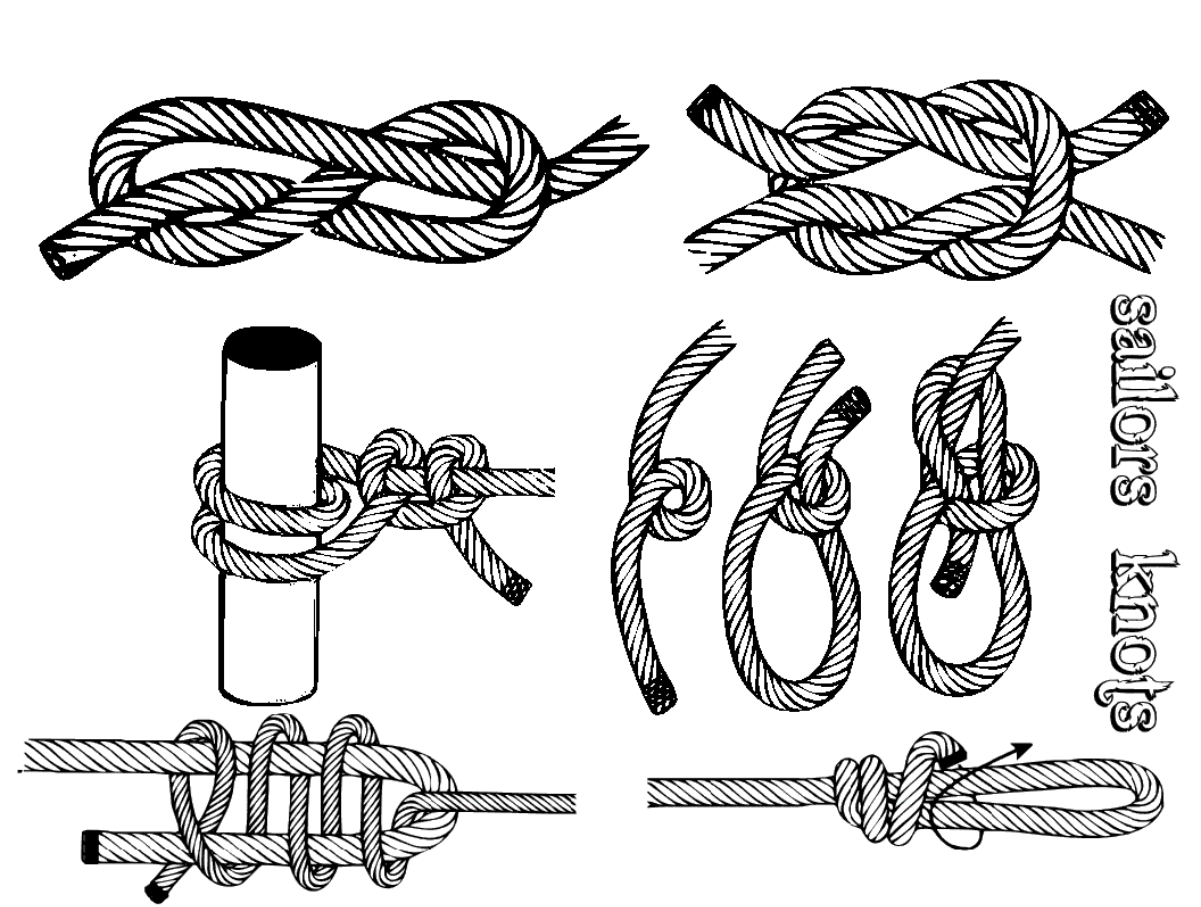
✓ Overall shape, deformations.
✓ “Clay mathematics”.
✓ mug $=$ doughnut
✓ Examples of applications:
1) Seven Bridges of Königsberg Problem;
2) knot theory.



3 Knots and braids

Knots and braids surround us:

✓ rope knots in sailing, mountaineering



3 Knots and braids

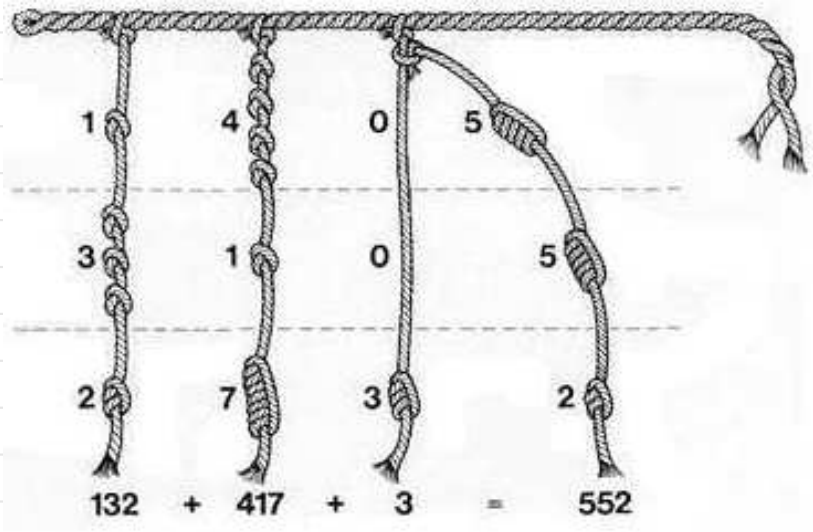
✓ ties

✓ hairstyles

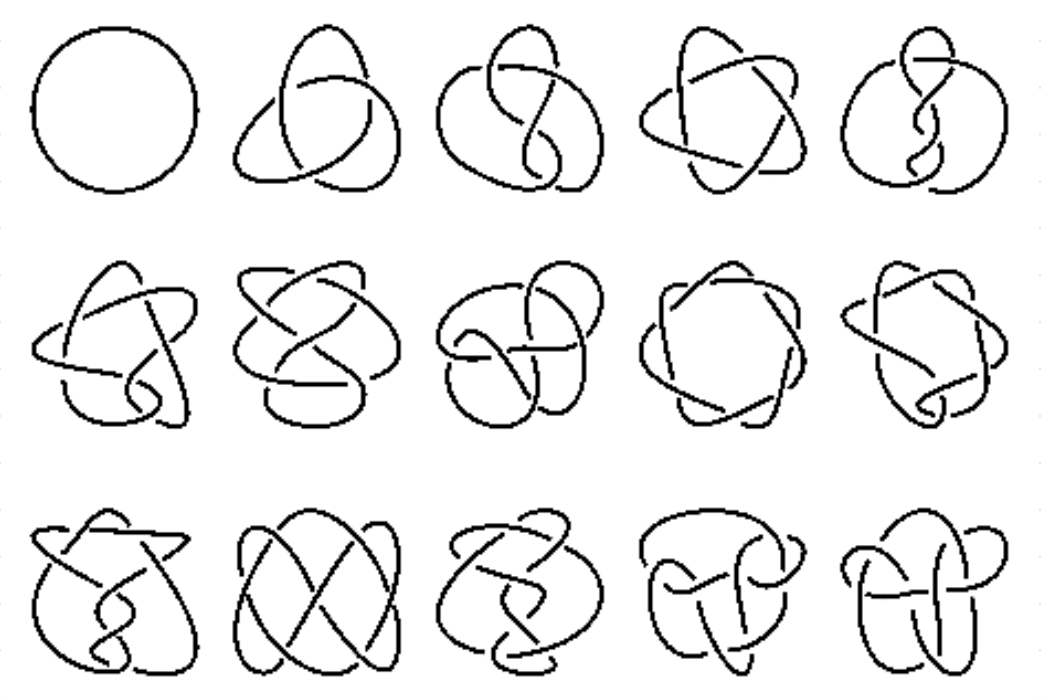
3 Knots and braids

✓ decoration, religion

✓ quipu: used by Incas
for recording information



Knots and braids in mathematics



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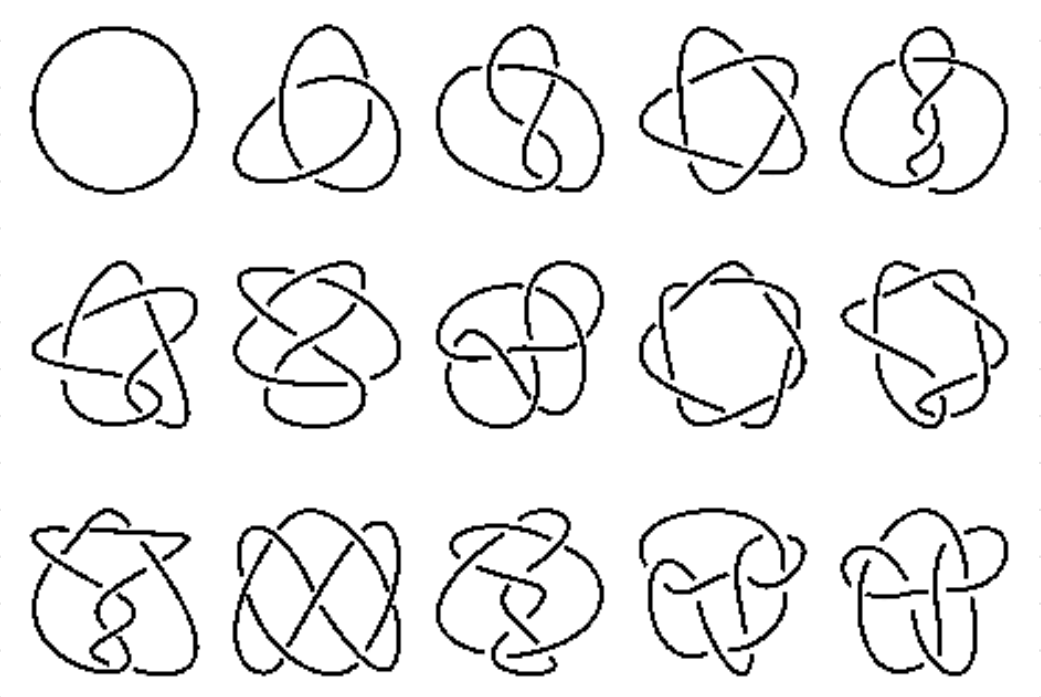
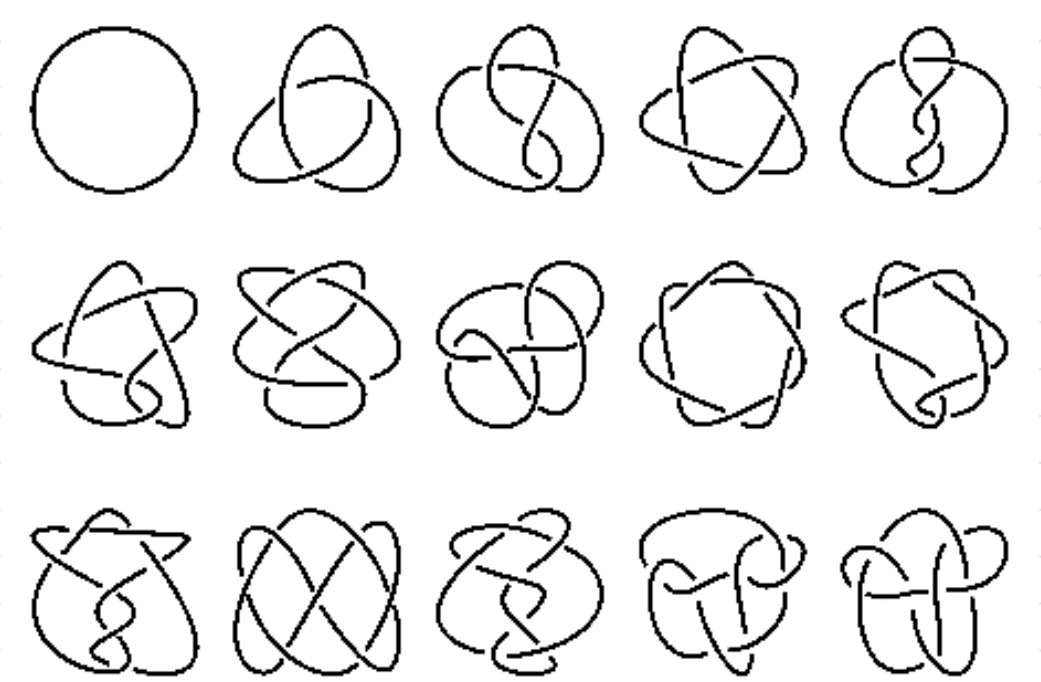


nature	many objects: different material, size, usage
mathematics	one abstraction: shape

Knots and braids in mathematics



nature	many objects: different material, size, usage
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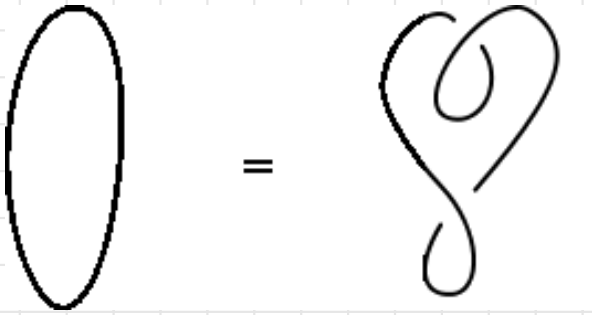
Mathematics begins when many objects are replaced with one abstraction.

Knots and braids in mathematics

In mathematics, knots and braids are considered up to deformation:



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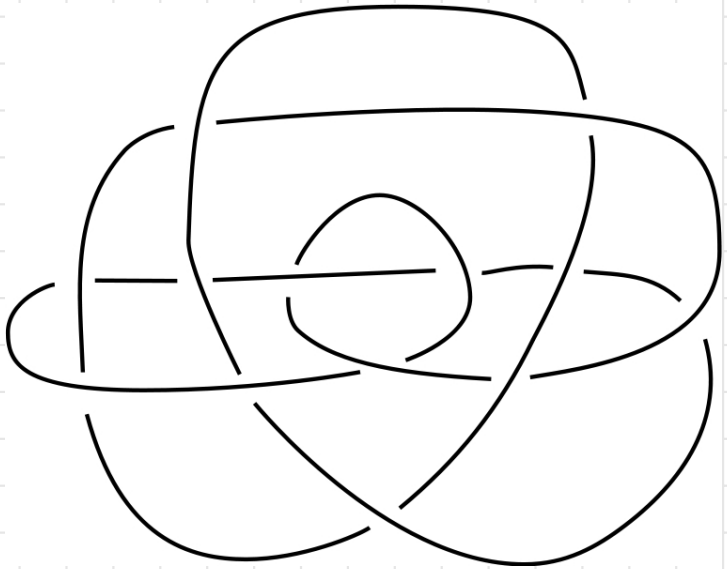

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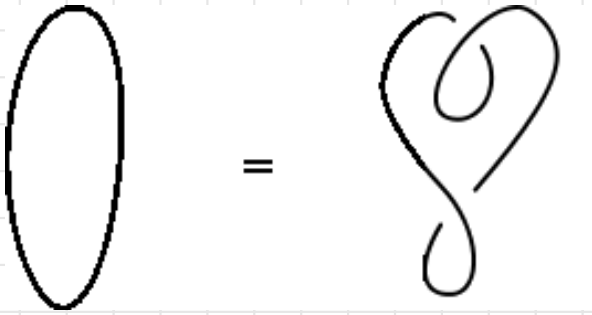

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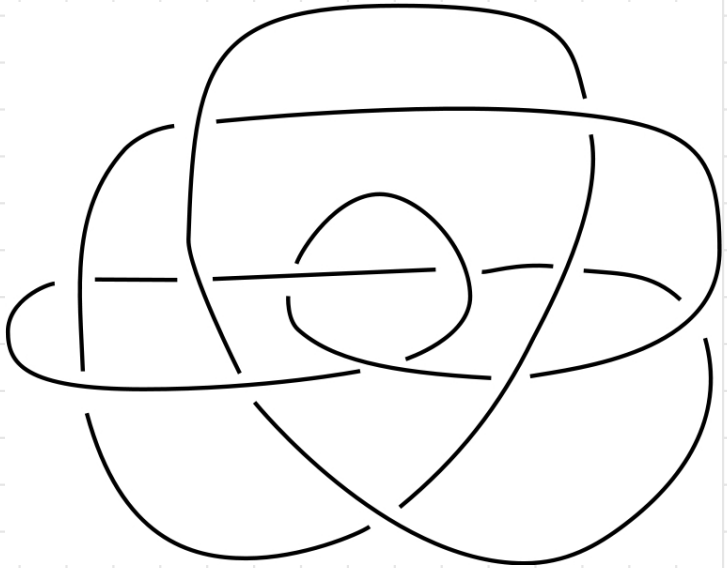

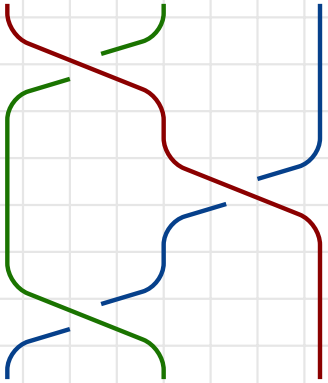

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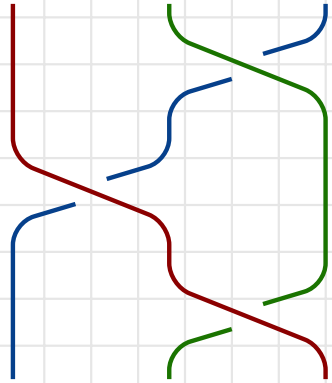
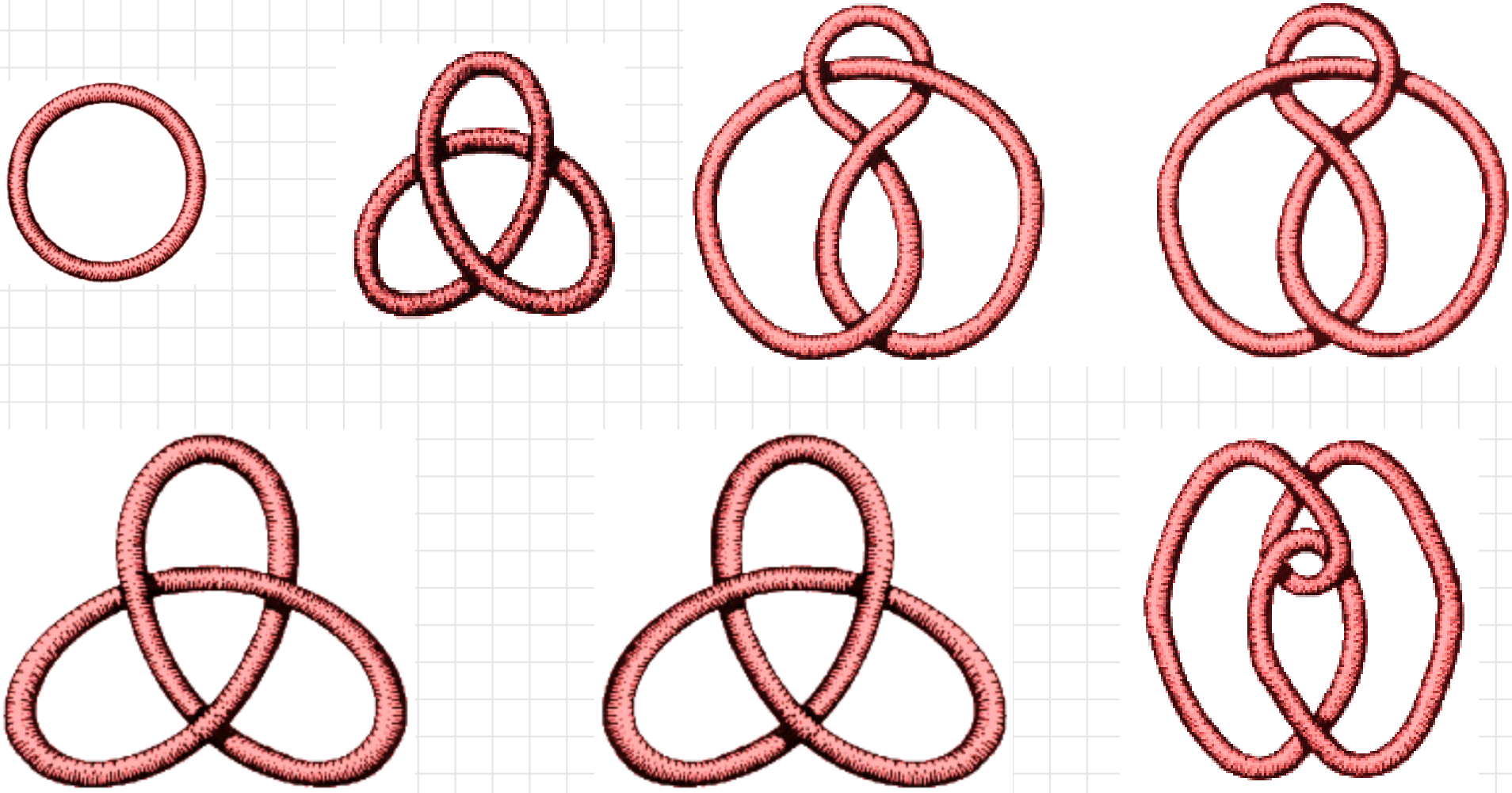


5 How to distinguish knots?



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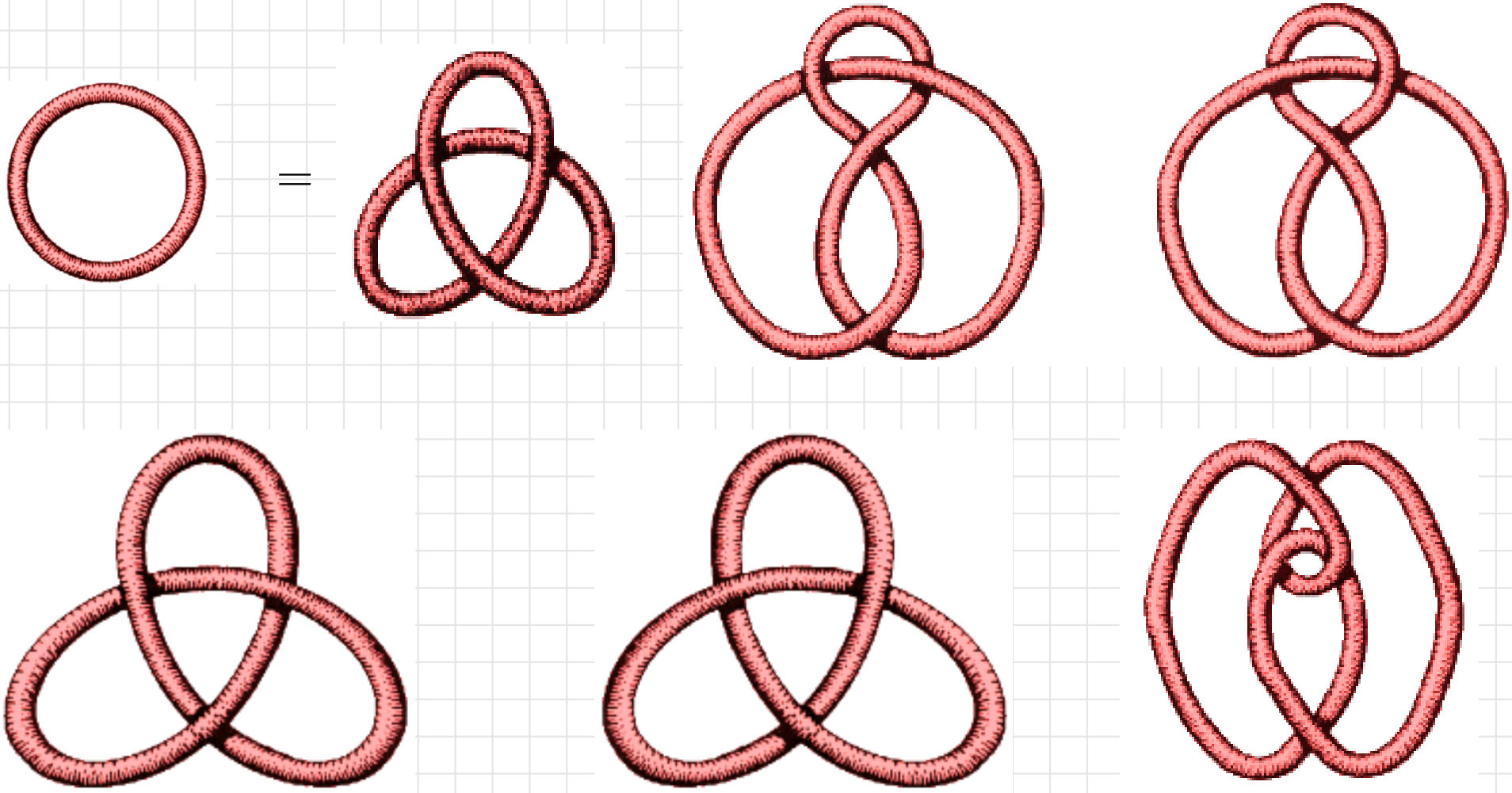

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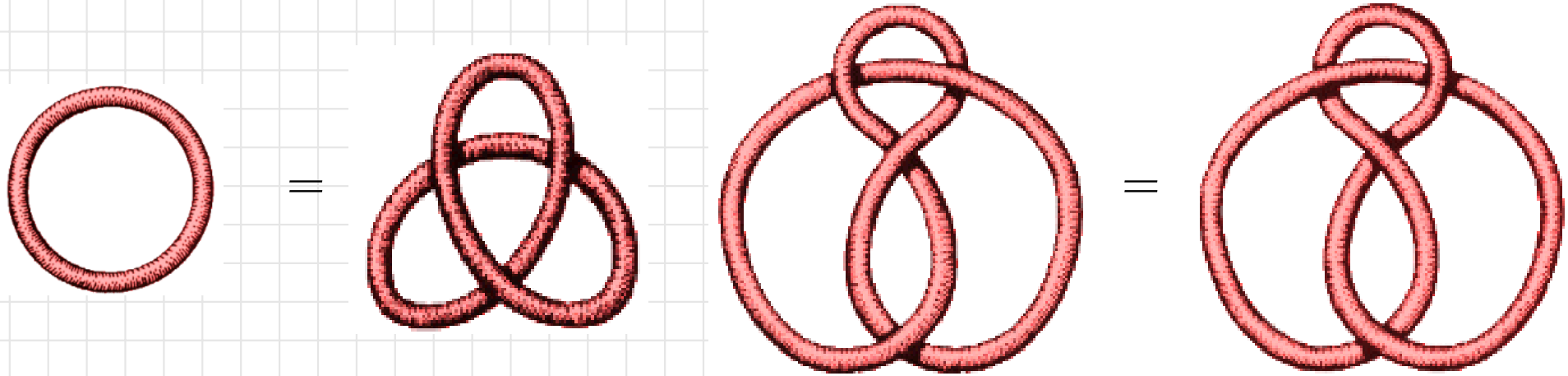

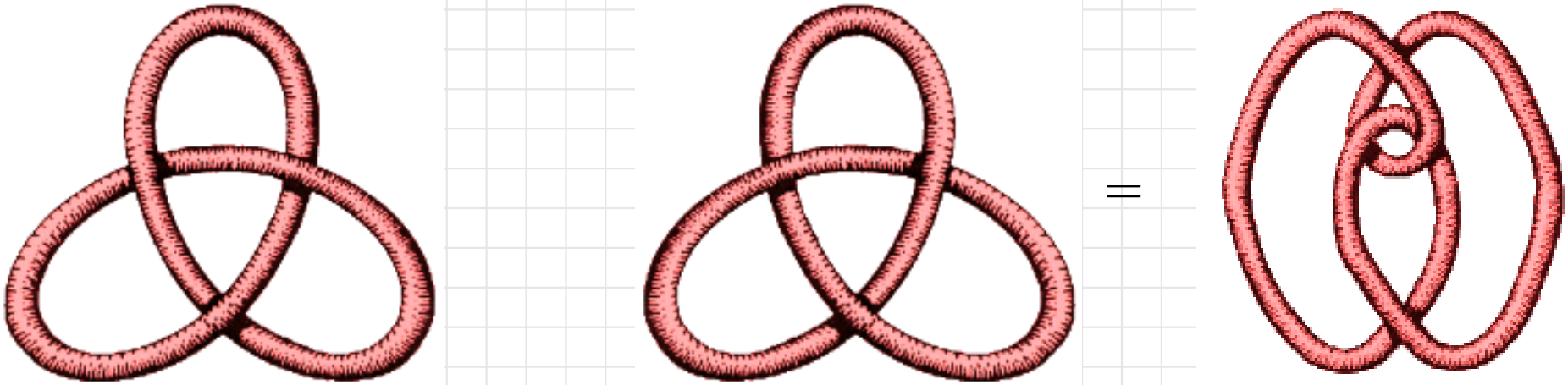

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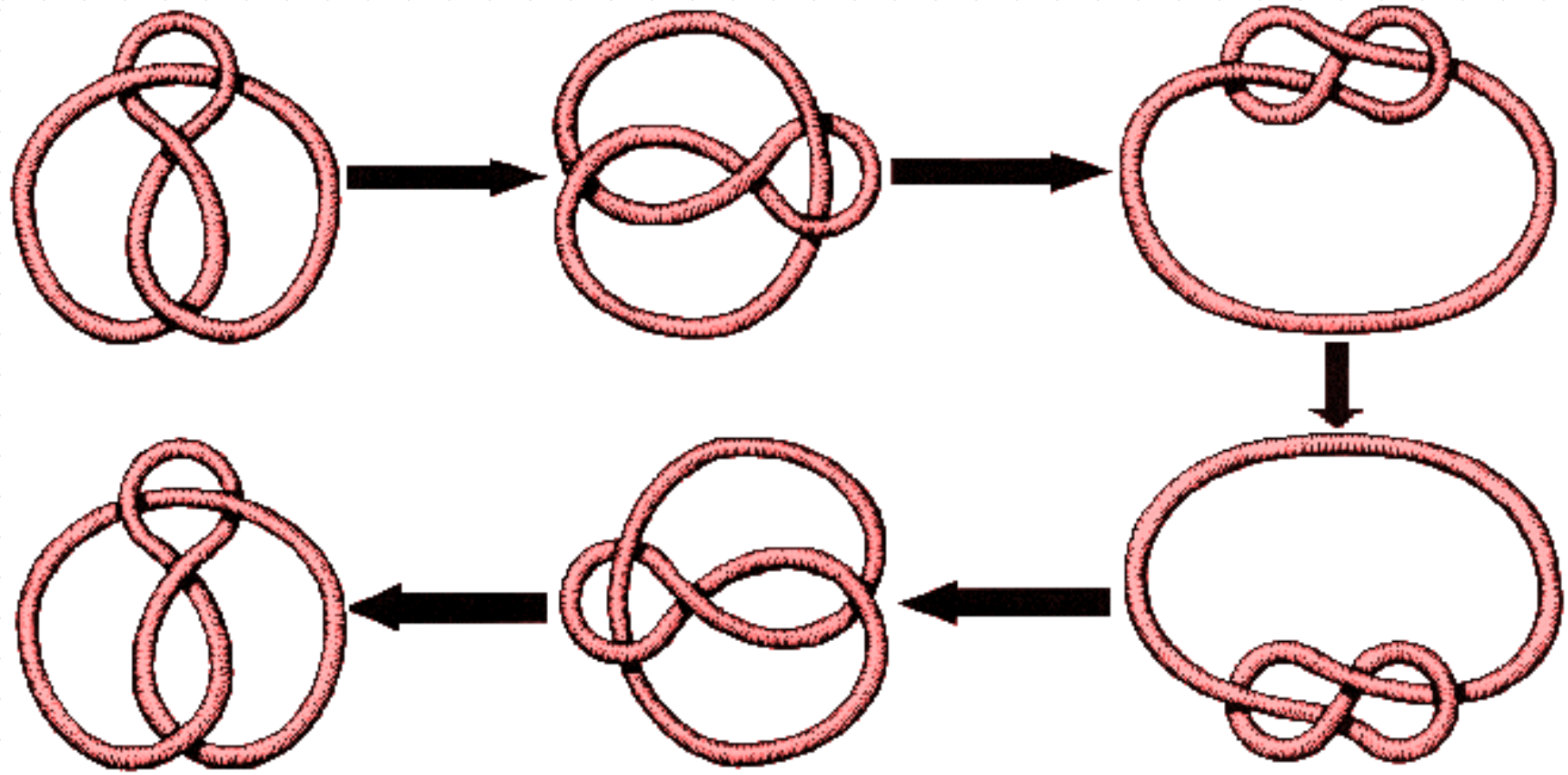
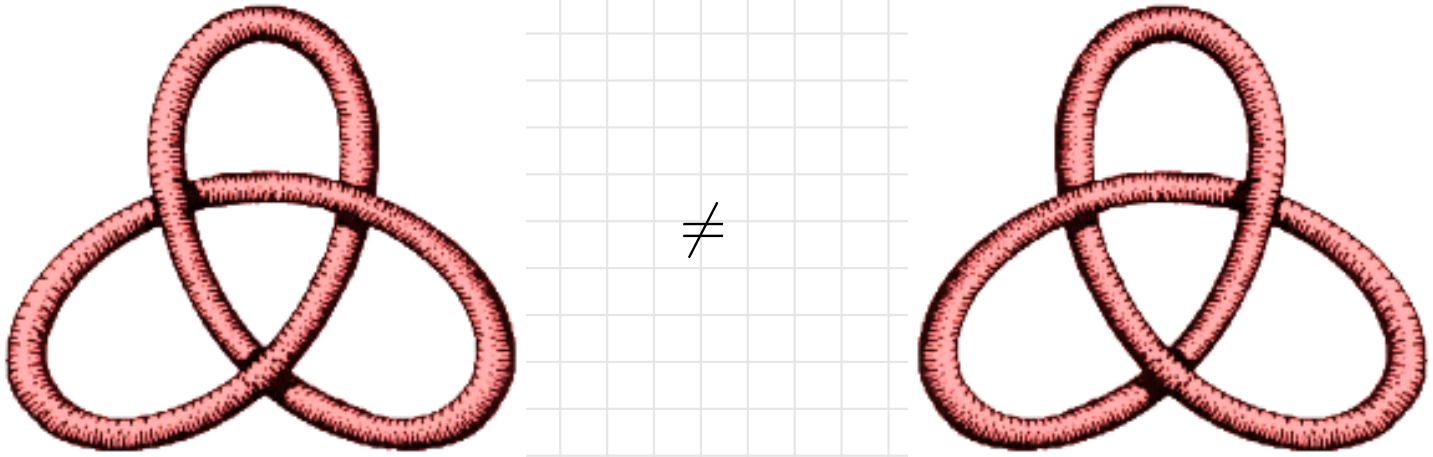
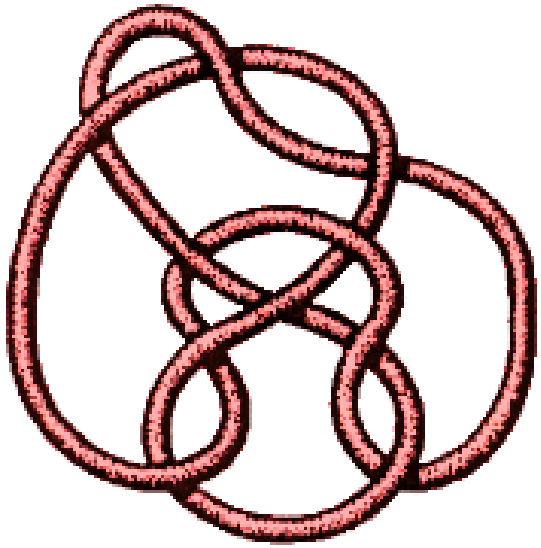


How to distinguish knots?

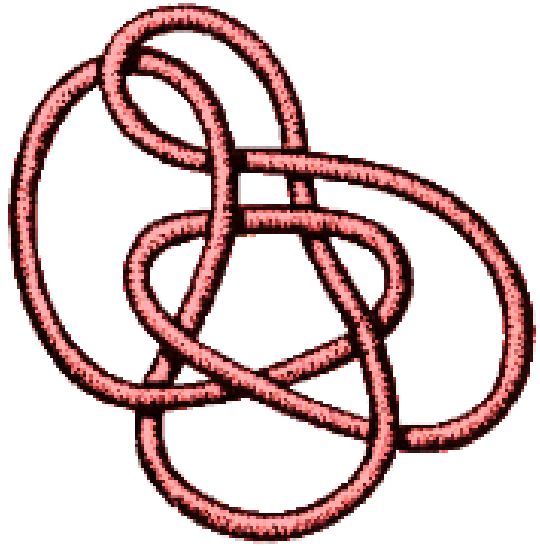
It is a difficult problem:



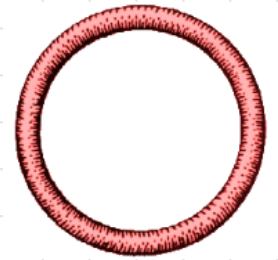
$\neq$



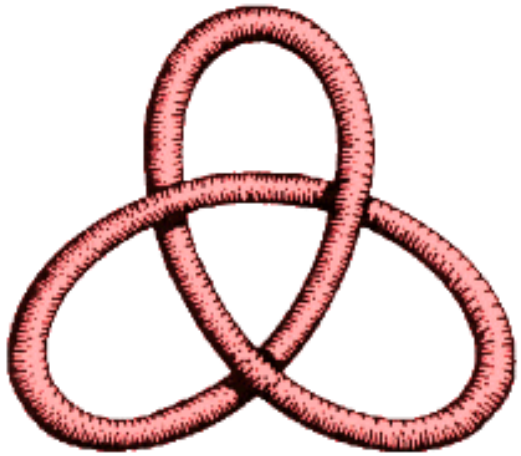
$=$



Knot invariants



$\overset{?}{\neq}$



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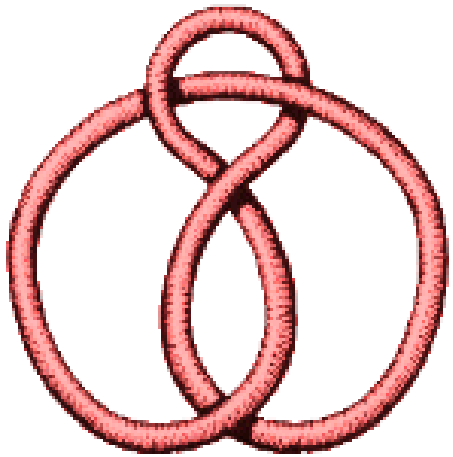
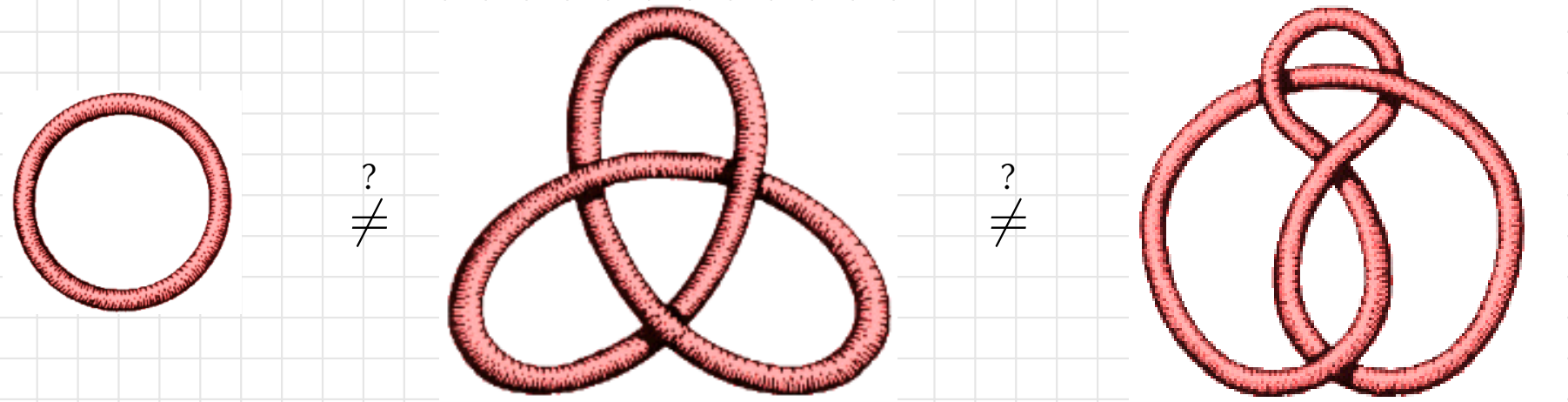


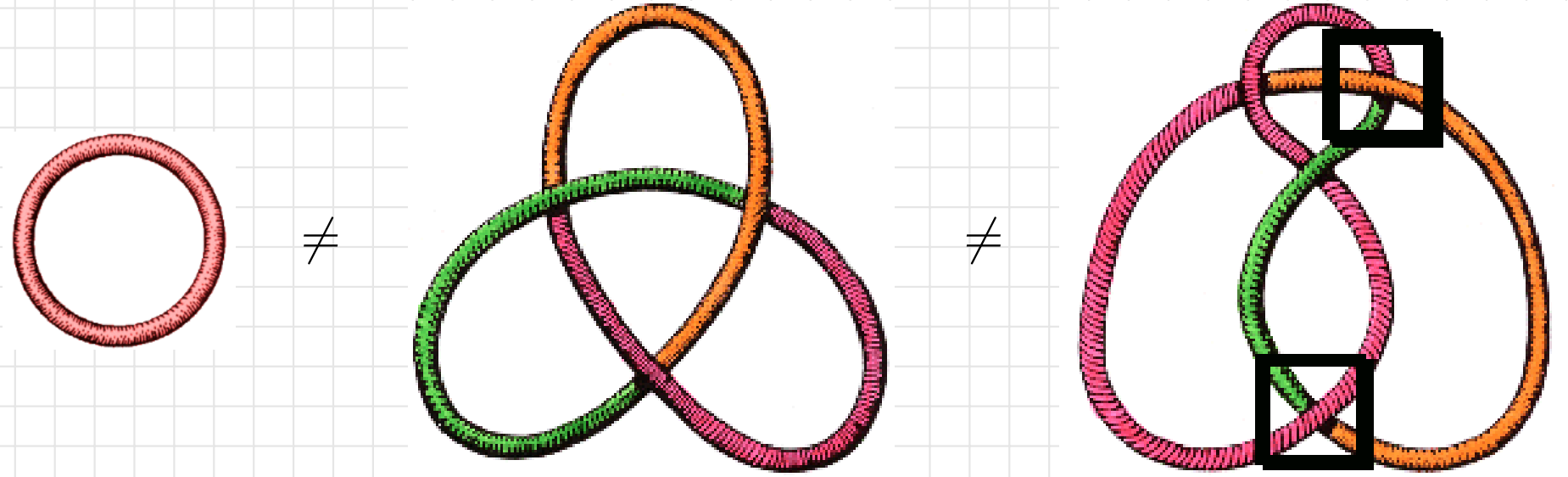
Knot invariants



Knot colourings: 3 colours

Rule: If a knot K′ is a deformation of a knot K, and K can be coloured by 3 colours, then K′ can be coloured by 3 colours as well.

Knot invariants



Can be coloured ?

no yes no

Knot theory: history

1867: Peter Tait experimented with **smoke rings**;

Knot theory: history

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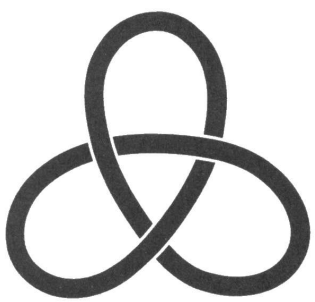
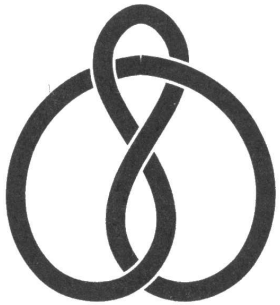
Lord Kelvin (Thomson): atoms = knotted tubes of ether



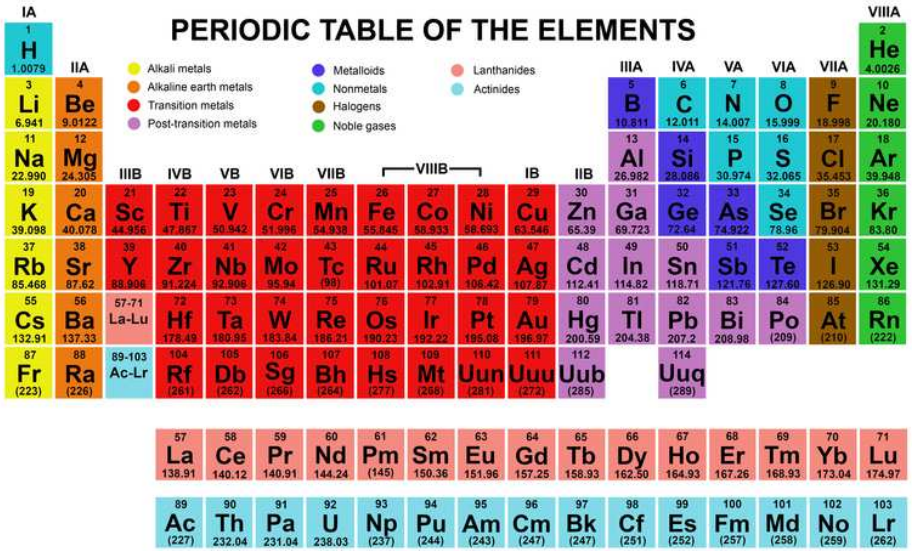
C (Carbon)



O (Oxygen)

H (Hydrogen)

7 Knot theory: history



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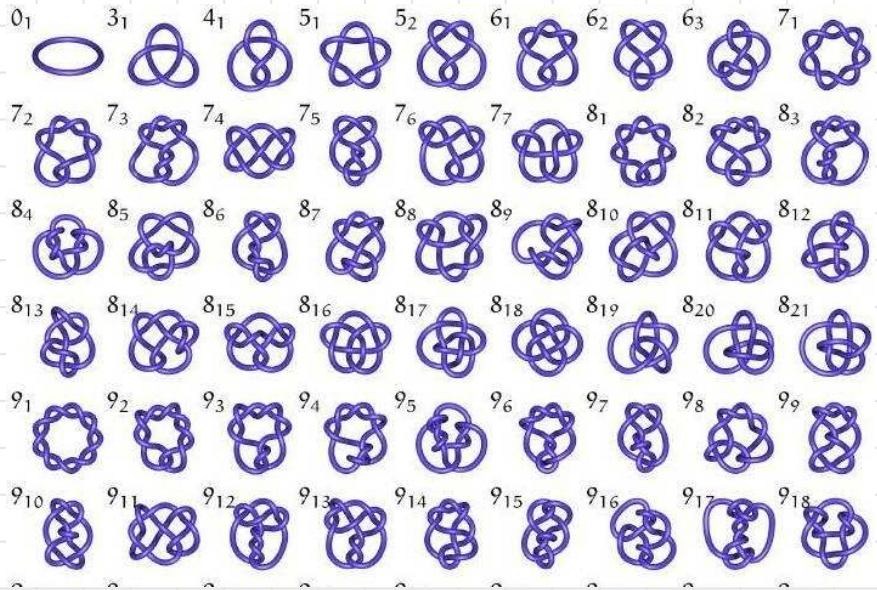

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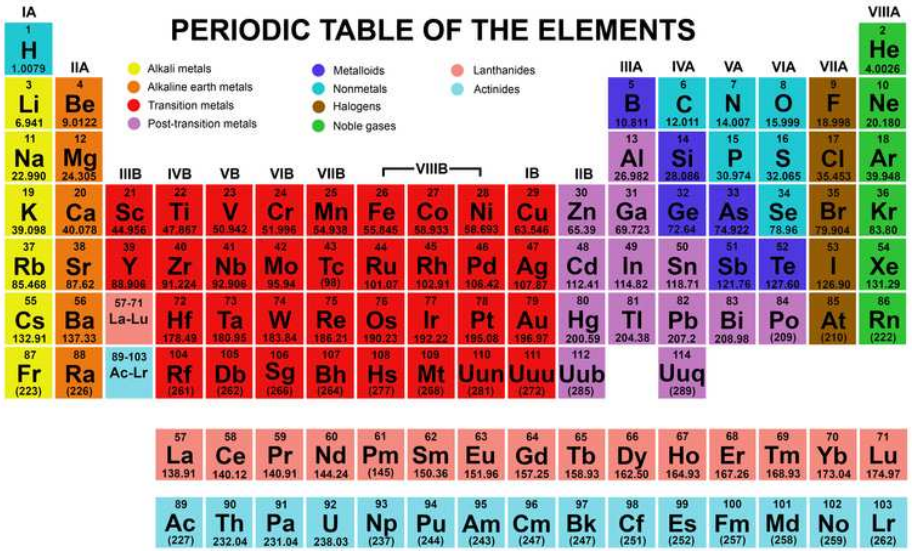

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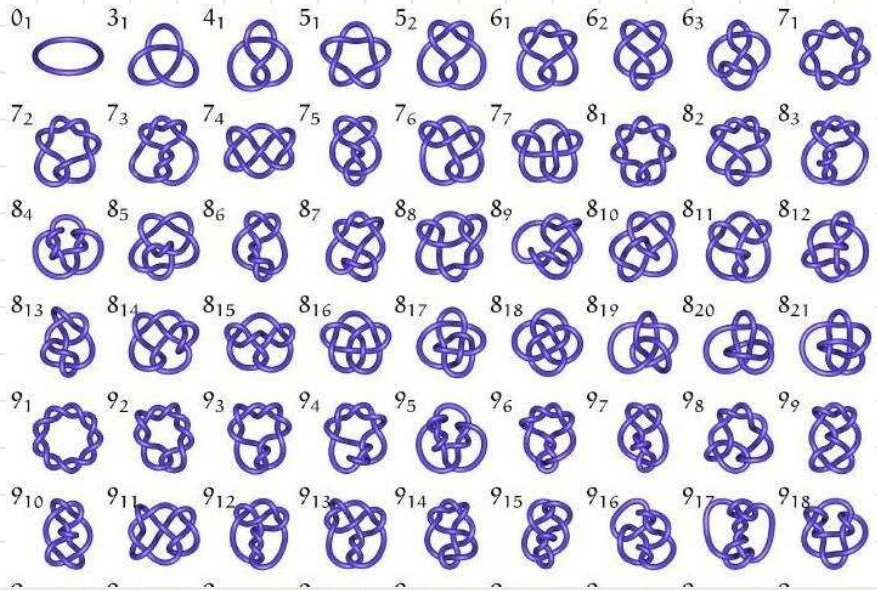
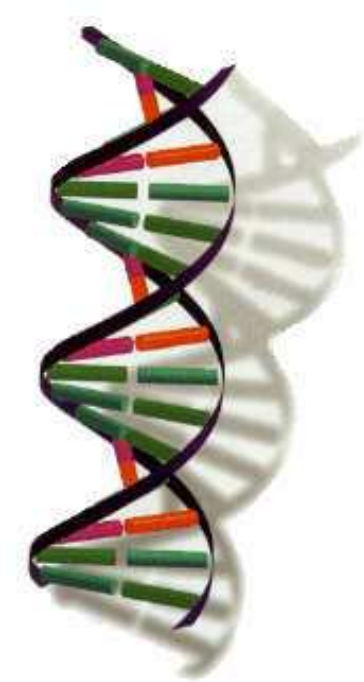


This is false!!!

Knot theory: applications

✓ **Biology**: DNA molecules.



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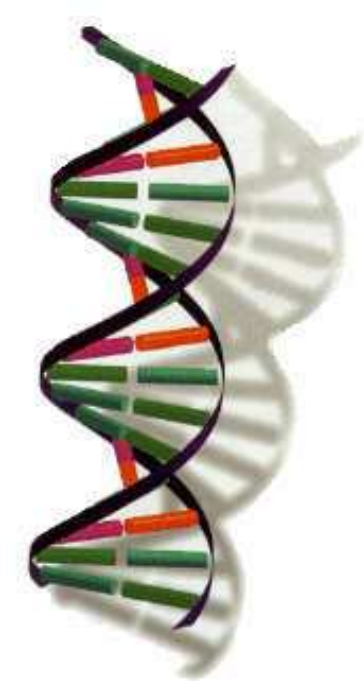
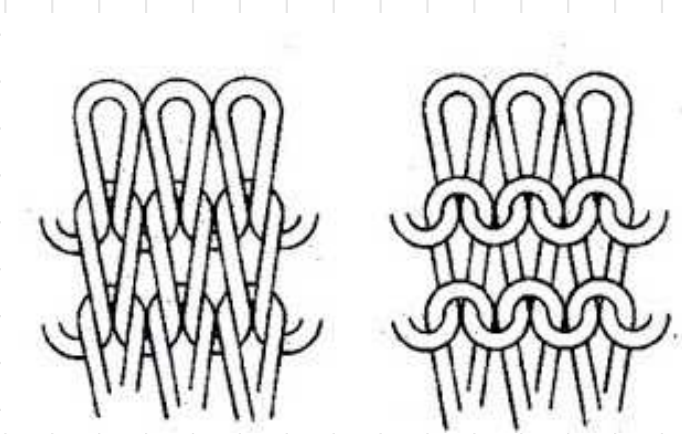
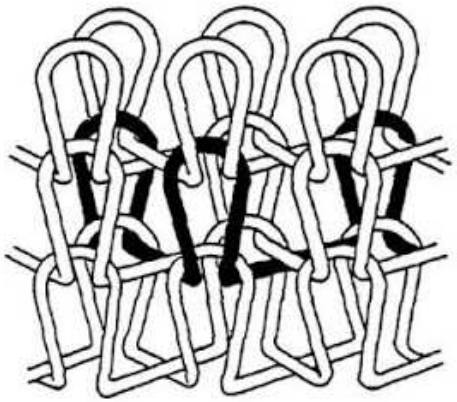


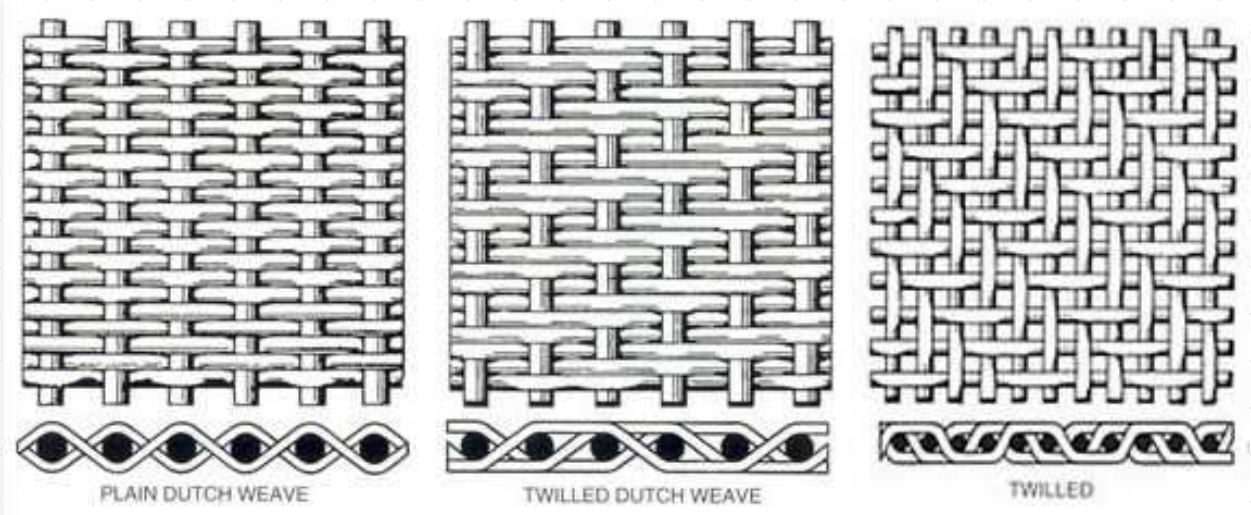
Some enzymes can cut, twist, and reconnect the DNA.
This changes the properties of the DNA.
Knot theory is used to detect the action of enzymes.

Knot theory: applications

✓ **Textile**: weaving patterns.







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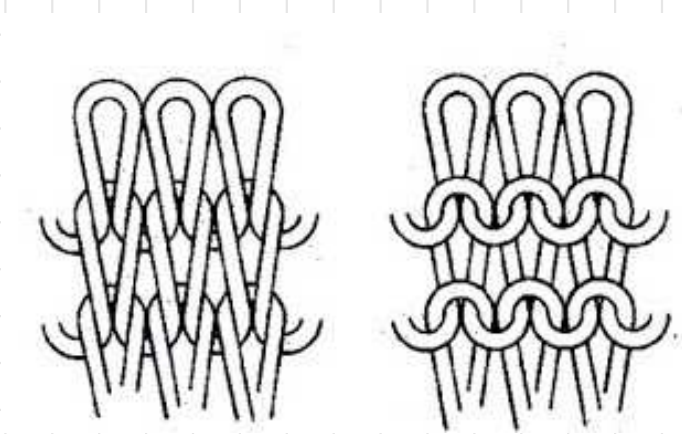

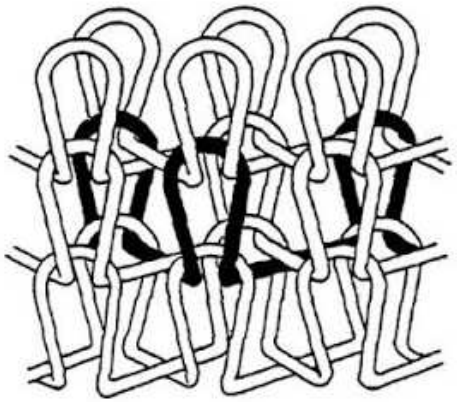

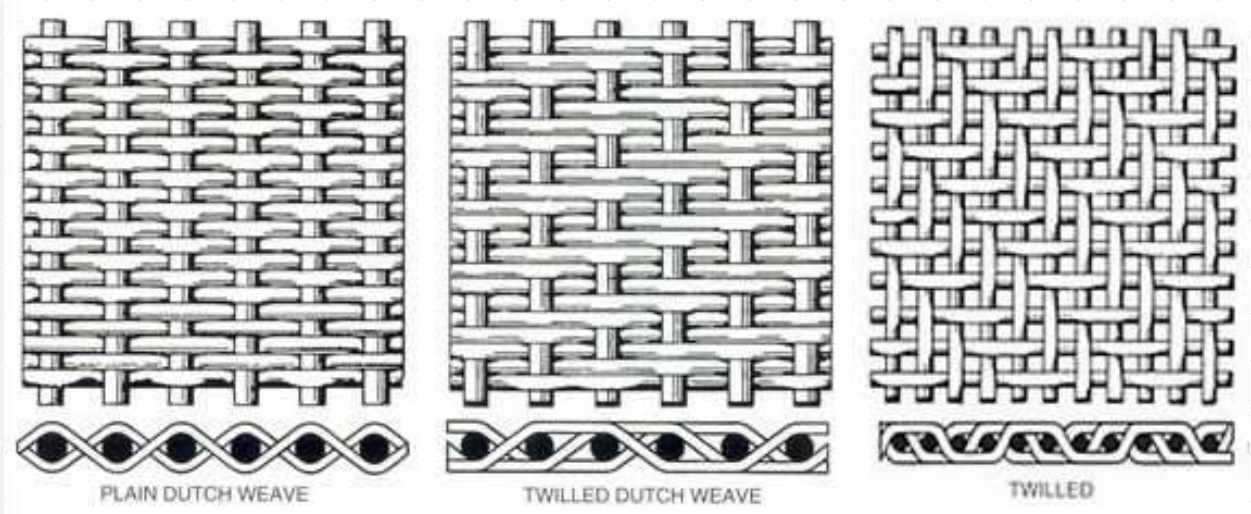

✓ **The shape of the universe** question: all 3-dimensional spaces can be encoded by knots.