

Geometry and Topology, or How Different a Mug and a Doughnut Really Are

Victoria LEBED, Research Fellow in Maths



1

What is mathematics?

Is it this?



1

What is mathematics?

And this?

Frank and Ernest



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1

What is mathematics?

Sometimes yes:

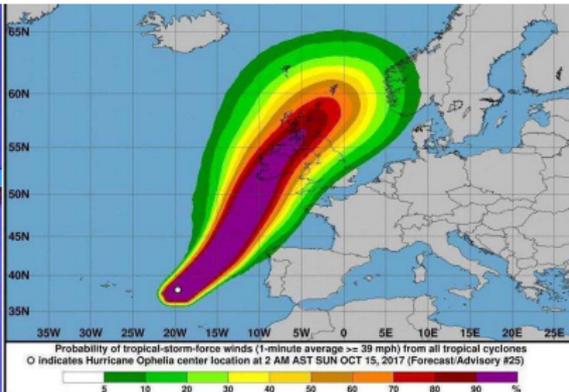


A screenshot from *Hidden Figures*.

1

What is mathematics?

But maths is also all of this:



Maths is about understanding **mechanisms** and seeing **patterns**, not only about computing! It involves **creativity**, and is sometimes closer to arts and philosophy than to sciences.

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Fortune 500's Most Valued Characteristics in an Employee:

Characteristics	1999	1970
Teamwork	1	10
Problem Solving	2	12
Interpersonal Skills	3	13
Oral Communication	4	4
Listening Skills	5	5
Personal Career Development	6	6
Creative Thinking	7	7
Leadership	8	8
Goal Setting/Motivation	9	9
Writing	10	1
Organizational Effectiveness	11	11
Computational Skills	12	2
Reading Skills	13	3

2

Mugs and doughnuts

Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.

Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.

2

Mugs and doughnuts

Geometry

- ✓ Distances, angles, curvature.
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Topology

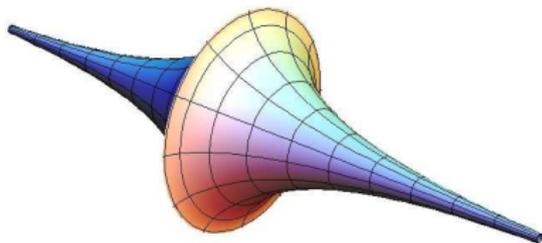
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Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.
- ✓ mug \neq doughnut
- ✓ Examples of applications:
 - 1) surveying;
 - 2) construction;
 - 3) astronomy;
 - 4) the shape of the universe.

Topology

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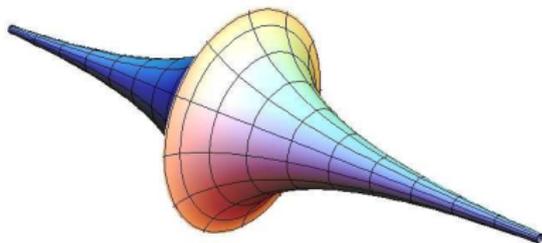


2

Mugs and doughnuts

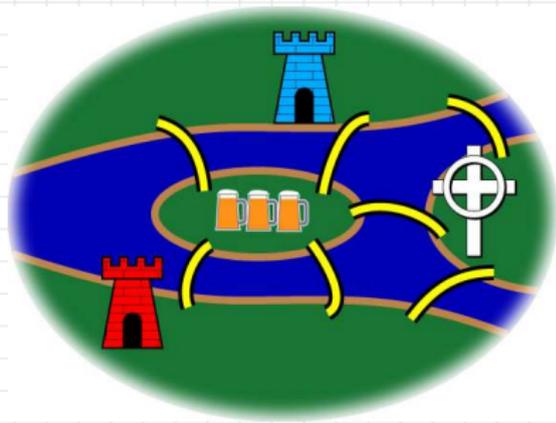
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Topology

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- ✓ Examples of applications:
 - 1) Seven Bridges of Königsberg Problem;

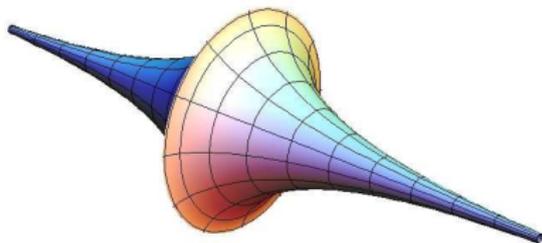


2

Mugs and doughnuts

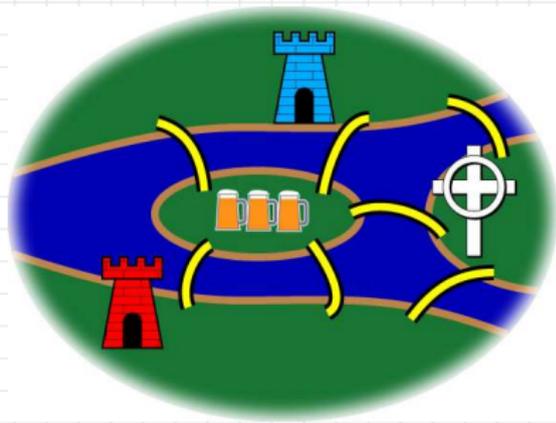
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Topology

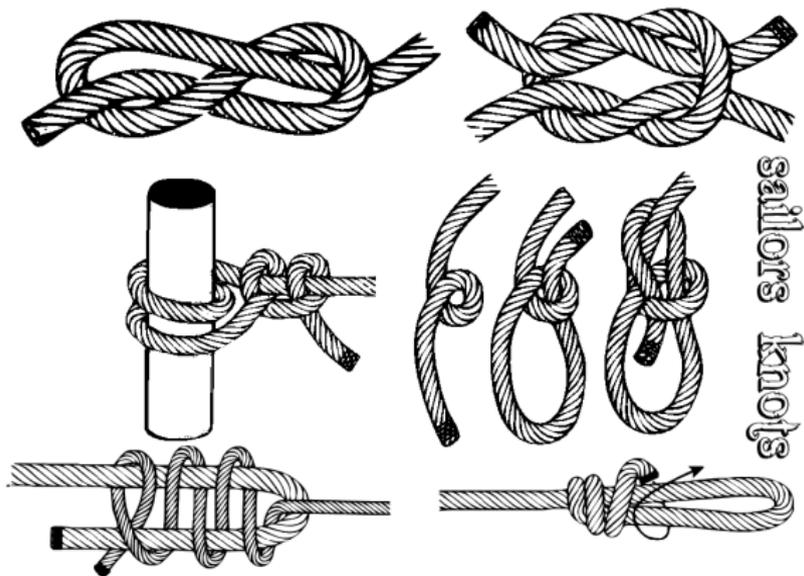
- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.
- ✓ mug = doughnut
- ✓ Examples of applications:
 - 1) Seven Bridges of Königsberg Problem;
 - 2) knot theory.



Knots and braids

Knots and braids surround us:

- ✓ rope knots in sailing, mountaineering



3

Knots and braids

✓ ties



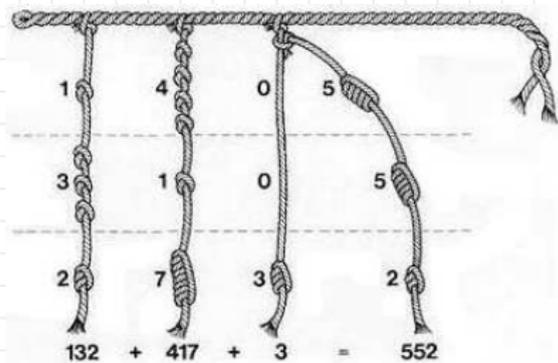
✓ hairstyles

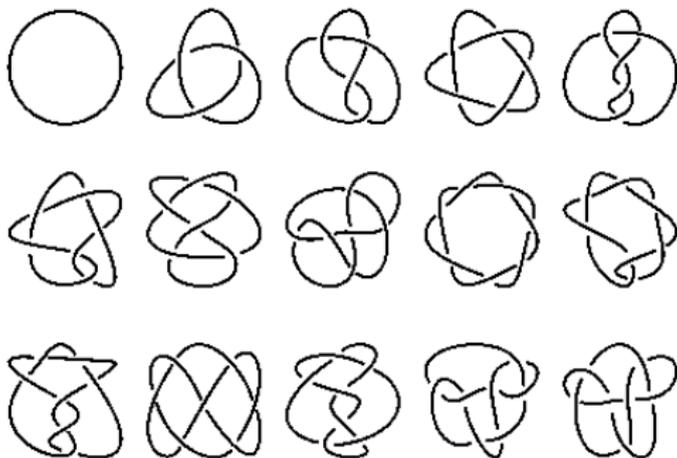


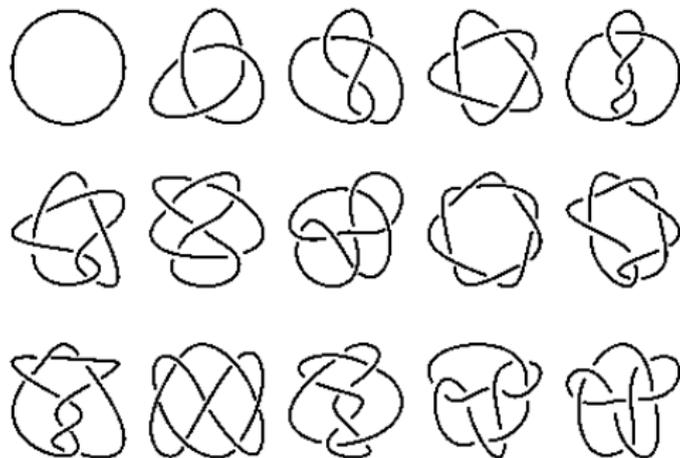
✓ decoration, religion



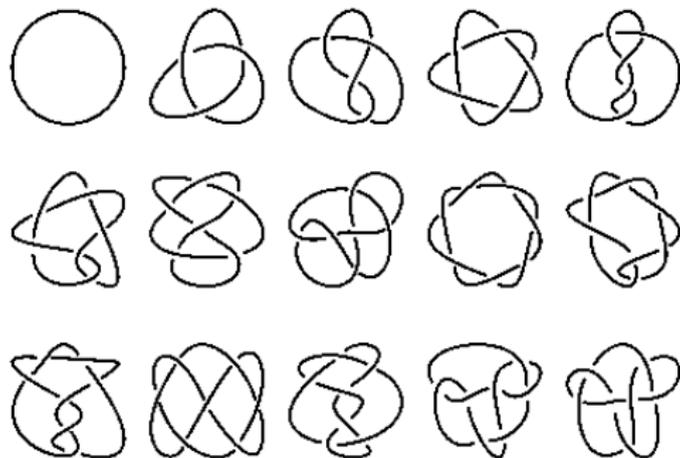
✓ quipu: used by Incas
for recording information







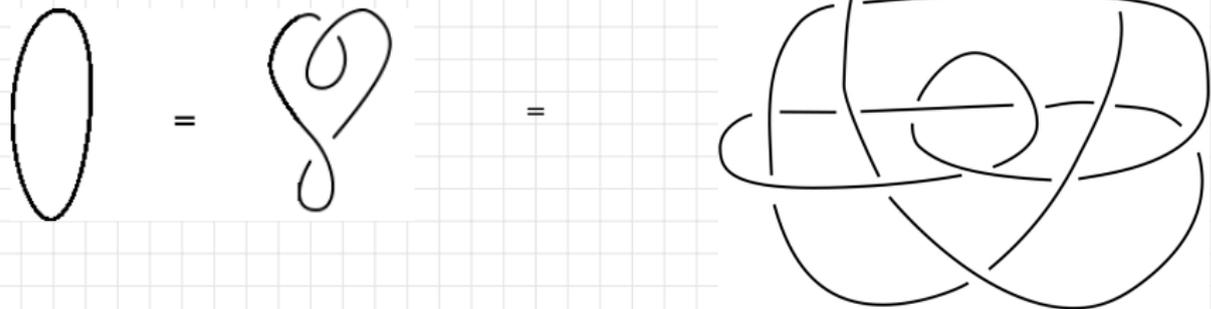
nature	many objects: different material, size, usage
mathematics	one abstraction: shape



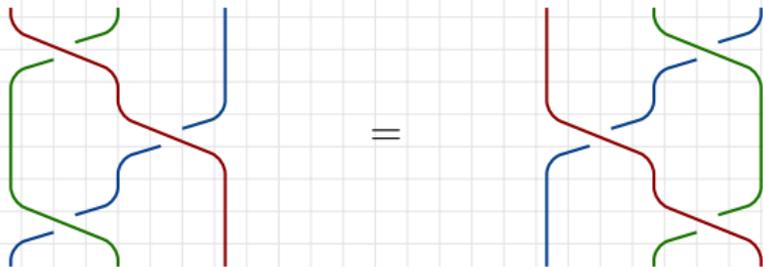
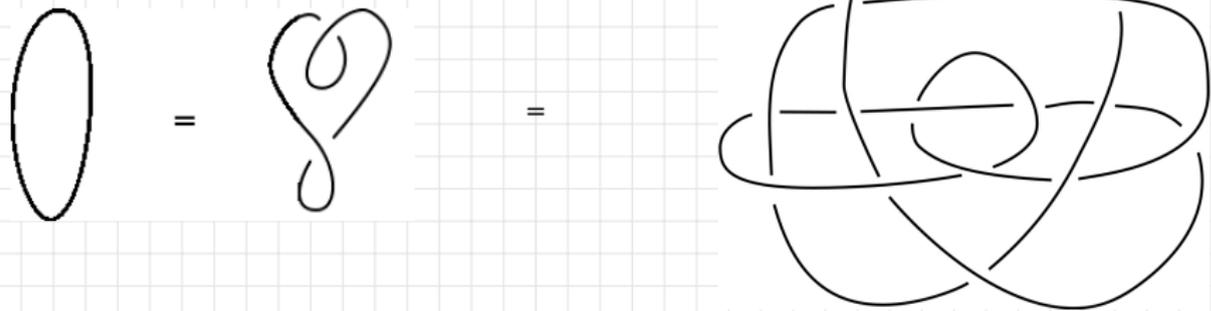
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Mathematics begins when many objects are replaced with one abstraction.

In mathematics, knots and braids are considered up to deformation:

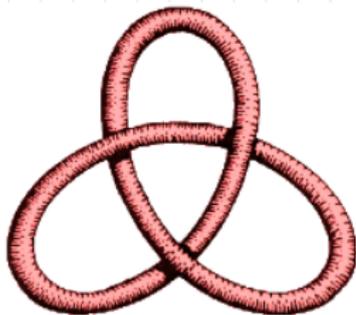
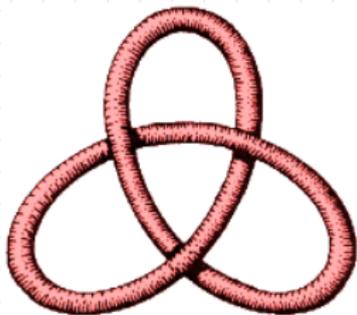


In mathematics, knots and braids are considered up to deformation:



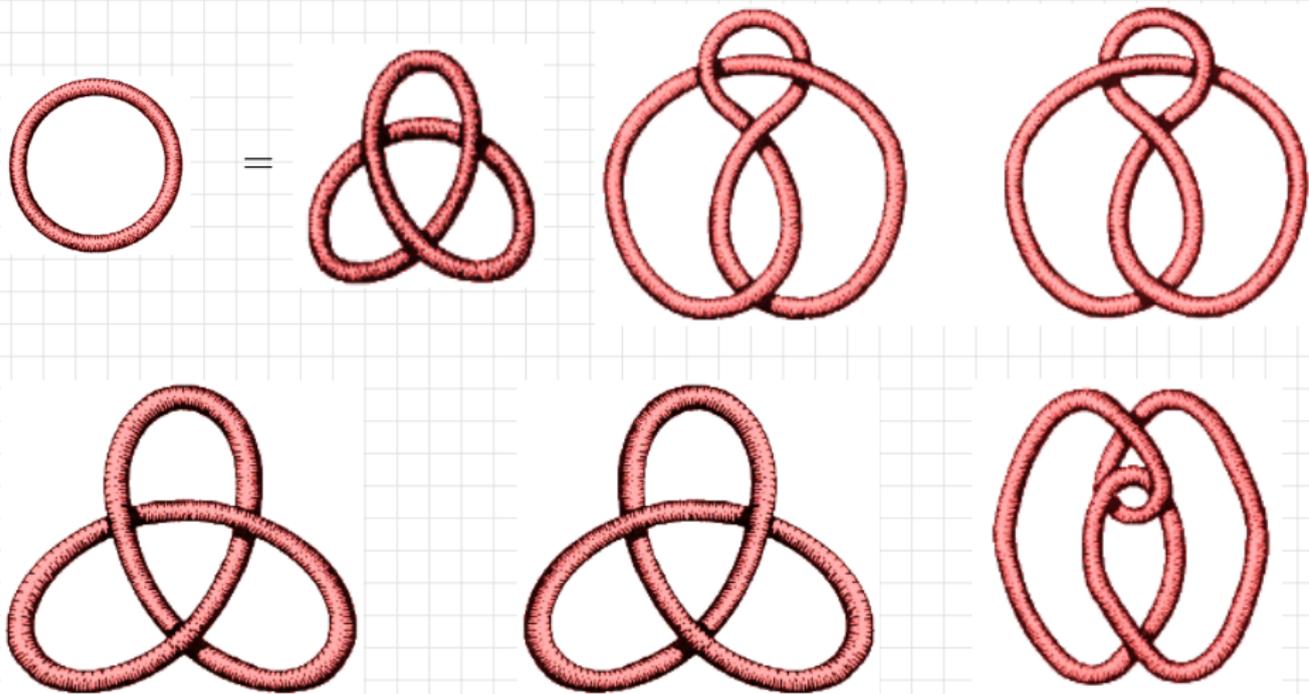
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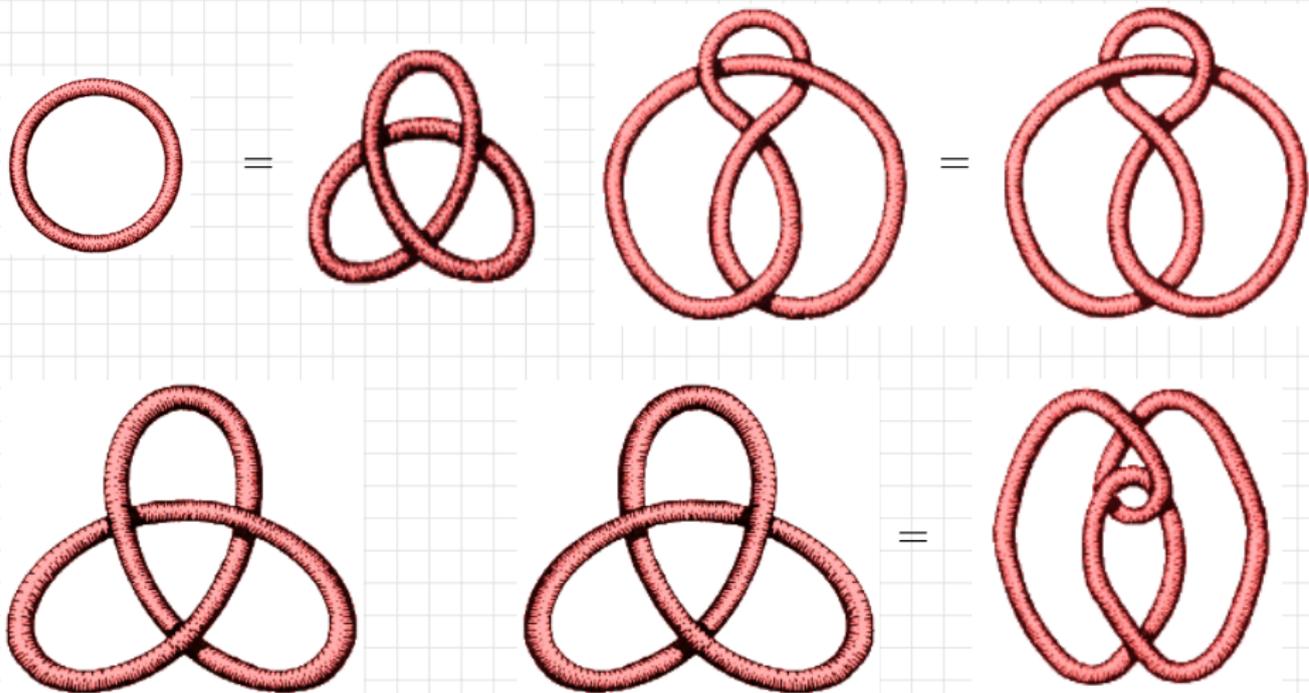
How to distinguish knots?



5

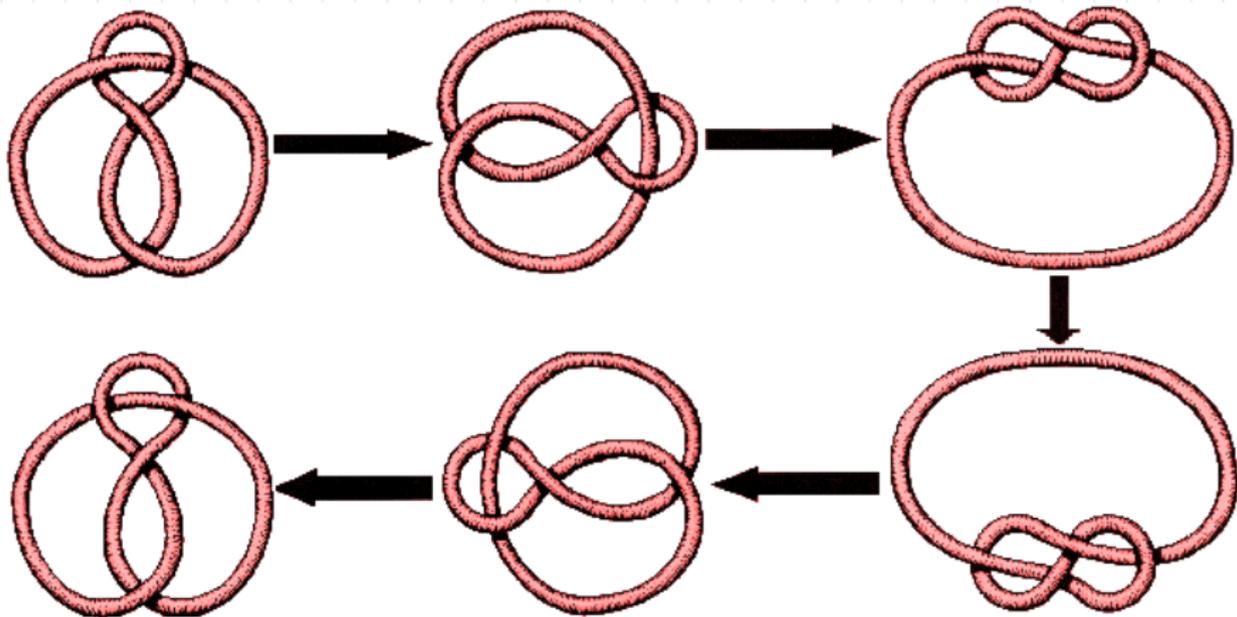
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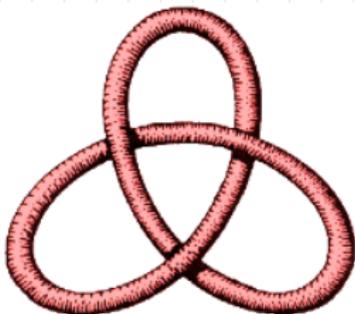
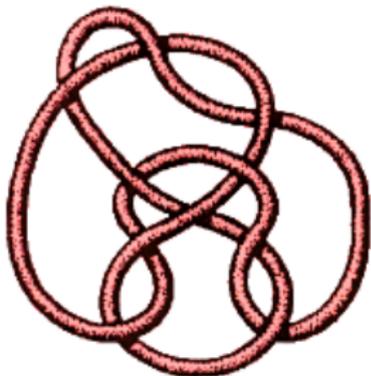
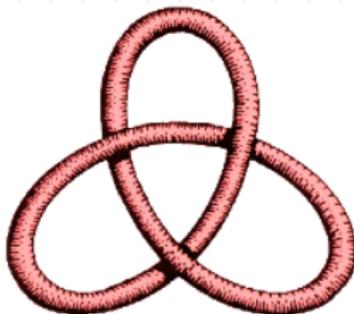
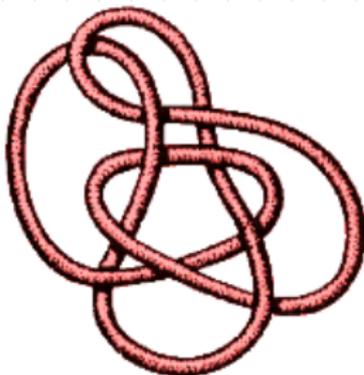
5

How to distinguish knots?



How to distinguish knots?

It is a difficult problem:

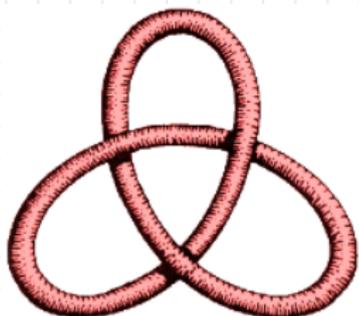
 \neq  $=$ 

6

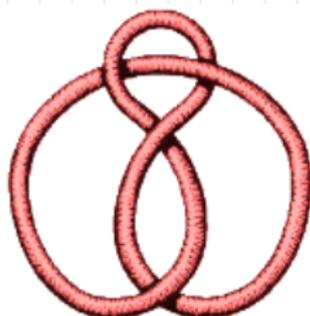
Knot invariants



?
 \neq

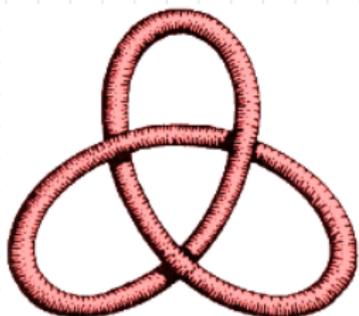
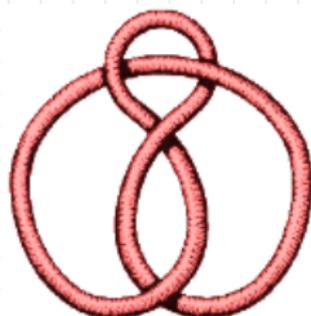


?
 \neq



6

Knot invariants

 $?$
 \neq  $?$
 \neq 

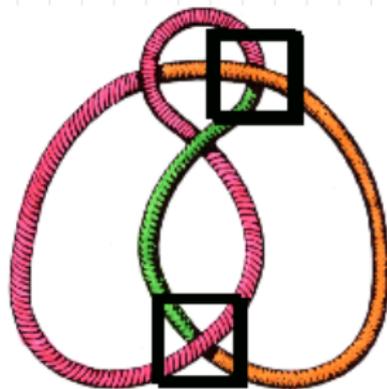
Knot colourings: 3 colours



Rule: If a knot K' is a deformation of a knot K , and K can be coloured by 3 colours, then K' can be coloured by 3 colours as well.

6

Knot invariants

 \neq  \neq 

Can be coloured ?

no

yes

no



Knot theory: history

1867: Peter Tait experimented with **smoke rings**;





Knot theory: history

1867: Peter Tait experimented with **smoke rings**;



Lord Kelvin (Thomson): atoms = knotted tubes of ether



C (Carbon)



O (Oxygen)



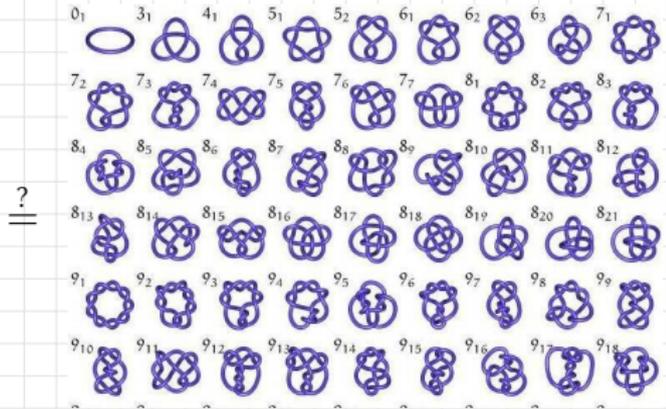
H (Hydrogen)

7 Knot theory: history

PERIODIC TABLE OF THE ELEMENTS

● Alkali metals ● Metalloids ● Lanthanides
● Alkaline earth metals ● Nonmetals ● Actinides
● Transition metals ● Halogens ● Noble gases
● Post transition metals ● Noble gases

1 H (1.0079)																	2 He (4.0026)
3 Li (6.941)	4 Be (9.0122)											5 B (10.811)	6 C (12.011)	7 N (14.007)	8 O (15.999)	9 F (18.998)	10 Ne (20.180)
11 Na (22.990)	12 Mg (24.305)											13 Al (26.982)	14 Si (28.086)	15 P (30.974)	16 S (32.06)	17 Cl (35.453)	18 Ar (39.948)
19 K (39.098)	20 Ca (40.078)	21 Sc (44.956)	22 Ti (47.88)	23 V (50.942)	24 Cr (51.996)	25 Mn (54.938)	26 Fe (55.845)	27 Co (58.933)	28 Ni (58.693)	29 Cu (63.546)	30 Zn (65.38)	31 Ga (69.723)	32 Ge (72.64)	33 As (74.922)	34 Se (78.96)	35 Br (79.904)	36 Kr (83.80)
37 Rb (85.468)	38 Sr (87.62)	39 Y (88.906)	40 Zr (91.224)	41 Nb (92.906)	42 Mo (95.94)	43 Tc (98.906)	44 Ru (101.07)	45 Rh (101.07)	46 Pd (106.32)	47 Ag (107.868)	48 Cd (112.41)	49 In (114.82)	50 Sn (118.71)	51 Sb (121.76)	52 Te (127.6)	53 I (126.905)	54 Xe (131.29)
55 Cs (132.91)	56 Ba (137.33)	57-71 La-Lu (138.905)	72 Hf (178.49)	73 Ta (180.948)	74 W (183.84)	75 Re (186.207)	76 Os (190.23)	77 Ir (192.22)	78 Pt (195.084)	79 Au (196.967)	80 Hg (200.59)	81 Tl (204.38)	82 Pb (207.2)	83 Bi (208.98)	84 Po ([209])	85 At ([210])	86 Rn ([222])
87 Fr ([223])	88 Ra ([226])	89-103 Ac-Lr ([227])	104 Rf ([261])	105 Db ([262])	106 Sg ([263])	107 Bh ([264])	108 Hs ([265])	109 Mt ([266])	110 Uun ([267])	111 Uuq ([268])	112 Uub ([269])	113 Uuq ([270])	114 Uuq ([271])	115 Uuq ([272])	116 Uuq ([273])	117 Uuq ([274])	118 Uuq ([276])
89 La (138.91)	90 Ce (140.12)	91 Pr (140.91)	92 Nd (144.24)	93 Pm ([145])	94 Sm (150.36)	95 Eu (151.96)	96 Gd (157.25)	97 Tb (158.93)	98 Dy (162.50)	99 Ho (164.93)	100 Er (167.26)	101 Tm (168.93)	102 Yb (173.04)	103 Lu (174.97)			
105 Ac ([227])	106 Th (232.04)	107 Pa (231.04)	108 U (238.03)	109 Np (237)	110 Pu (244)	111 Am (243)	112 Cm (247)	113 Bk (247)	114 Cf (251)	115 Es (252)	116 Fm (257)	117 Md (258)	118 No (259)	119 Lr (262)			



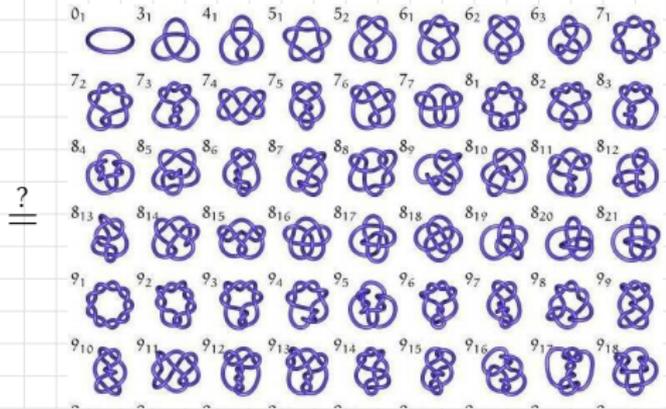
7 Knot theory: history

PERIODIC TABLE OF THE ELEMENTS

Legend:

- Alkali metals (Yellow)
- Alkaline earth metals (Orange)
- Transition metals (Red)
- Post transition metals (Purple)
- Metalloids (Green)
- Nonmetals (Blue)
- Halogens (Light Blue)
- Noble gases (Light Green)
- Lanthanides (Pink)
- Actinides (Light Purple)

1 H 1.00794	2 He 4.002602																	18 Ar 39.948	19 K 39.0983	20 Ca 40.078											80 Hg 200.59	81 Tl 204.38	82 Pb 207.2	83 Bi 208.98	84 Po [209]	85 At [210]	86 Rn [222]																			
3 Li 6.941	4 Be 9.0122																	35 Br 79.904	36 Kr 83.80	37 Rb 85.468	38 Sr 87.62	39 Y 88.906	40 Zr 91.224	41 Nb 92.906	42 Mo 95.94	43 Tc [98]	44 Ru 101.07	45 Rh 101.07	46 Pd 106.36	47 Ag 107.868	48 Cd 112.411	49 In 114.818	50 Sn 118.710	51 Sb 121.757	52 Te 127.60	53 I 126.905	54 Xe 131.29	55 Cs 132.905	56 Ba 137.327	57-70 La-Lu [138.905]	71 Lu 174.967	72 Hf 178.49	73 Ta 180.948	74 W 183.84	75 Re 186.207	76 Os 190.23	77 Ir 192.222	78 Pt 195.084	79 Au 196.967	80 Hg 200.59	81 Tl 204.38	82 Pb 207.2	83 Bi 208.98	84 Po [209]	85 At [210]	86 Rn [222]
57 La 138.905	58 Ce 140.12	59 Pr 140.908	60 Nd 144.24	61 Pm [145]	62 Sm 150.36	63 Eu 151.964	64 Gd 157.25	65 Tb 158.925	66 Dy 162.50	67 Ho 164.930	68 Er 167.259	69 Tm 168.934	70 Yb 173.045	71 Lu 174.967	87 Fr [223]	88 Ra [226]	89-103 Ac-Lr [227]	104 Rf [261]	105 Db [262]	106 Sg [263]	107 Bh [264]	108 Hs [265]	109 Mt [266]	110 Uun [267]	111 Uuq [268]	112 Uub [269]	113 Uuq [270]	114 Uuq [271]	115 Uuq [272]	116 Uuq [273]	117 Uuq [274]	118 Uuq [275]	119 Uuq [276]	120 Uuq [277]																						

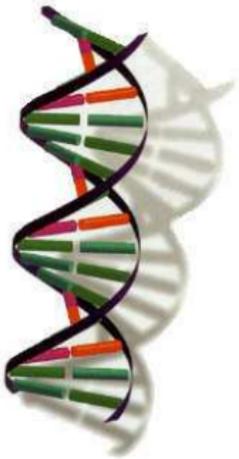


This is false!!!

8

Knot theory: applications

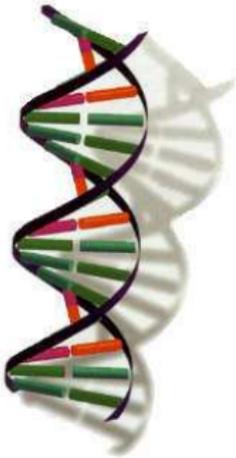
✓ **Biology:** DNA molecules.



8

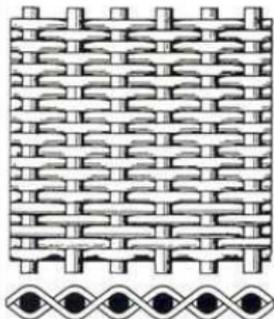
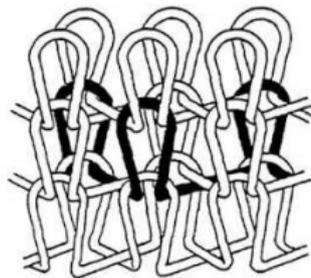
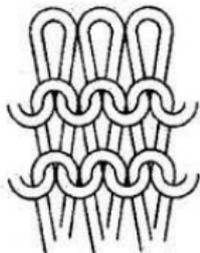
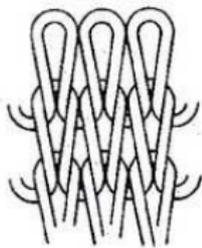
Knot theory: applications

✓ **Biology:** DNA molecules.

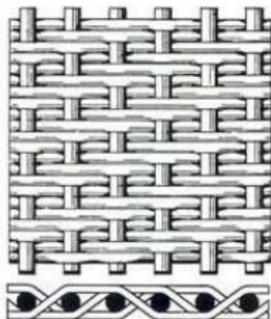


Some enzymes can cut, twist, and reconnect the DNA.
This changes the properties of the DNA.
Knot theory is used to detect the action of enzymes.

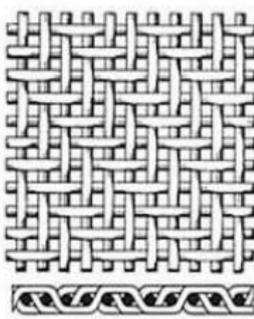
✓ **Textile:** weaving patterns.



PLAIN DUTCH WEAVE

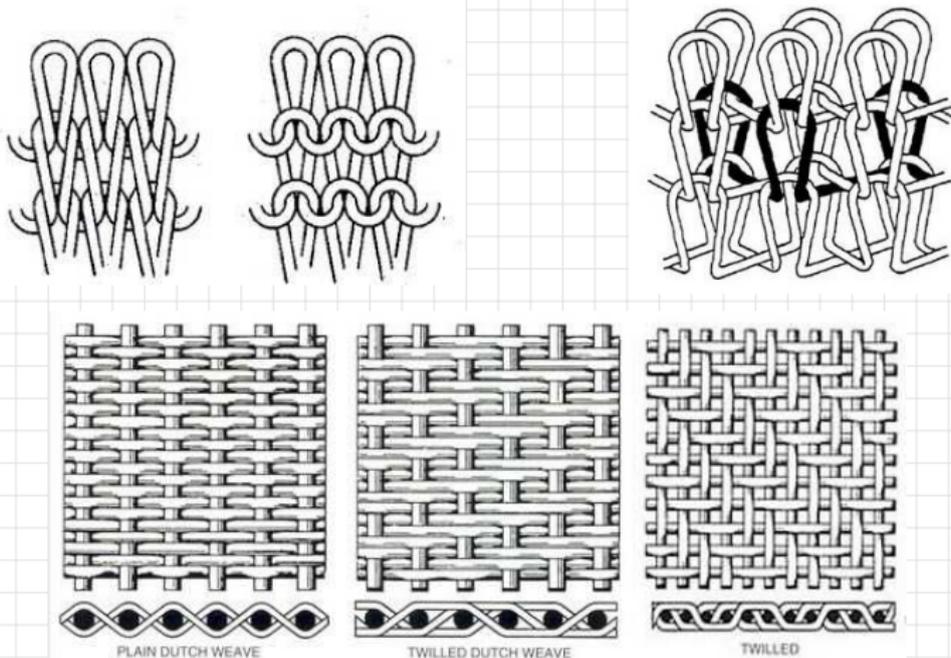


TWILLED DUTCH WEAVE



TWILLED

✓ **Textile:** weaving patterns.



✓ **The shape of the universe** question: all 3-dimensional spaces can be encoded by knots.