

# Geometry and Topology, or How Different a Mug and a Doughnut Really Are

**Victoria LEBED,** Research Fellow in Maths



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# What is mathematics?

Is it this?



1

# What is mathematics?

And this?

## Frank and Ernest

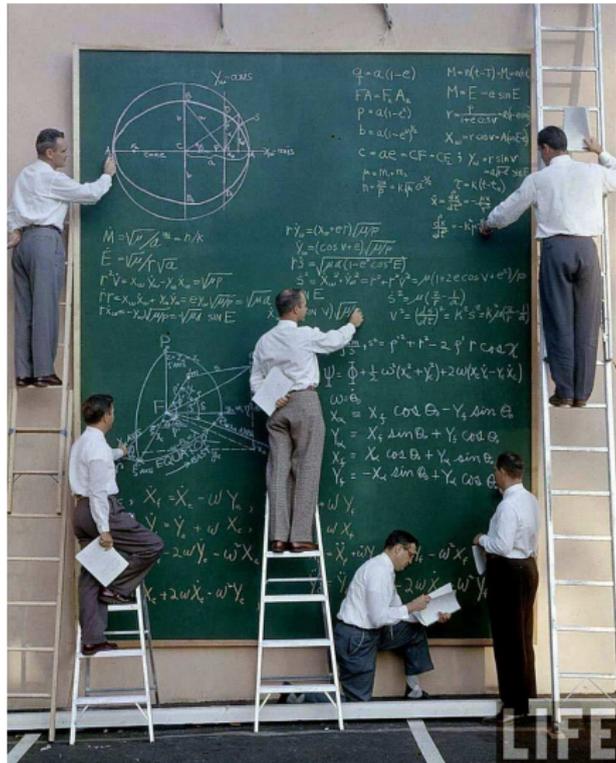


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# What is mathematics?

Sometimes yes:



NASA scientists with their board of calculations, 1961, *Life* magazine.

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# What is mathematics?

Sometimes yes:

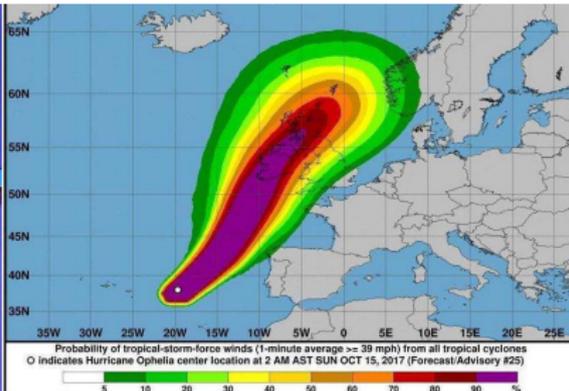


A screenshot from *Hidden Figures*.

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# What is mathematics?

But maths is also all of this:



Maths is about understanding **mechanisms** and seeing **patterns**, not only about computing! It involves **creativity**, and is sometimes closer to arts and philosophy than to sciences.

Maths is about understanding **mechanisms** and seeing **patterns**, not only about computing! It involves **creativity**, and is sometimes closer to arts and philosophy than to sciences.

Fortune 500's Most Valued Characteristics in an Employee:

Characteristics	1999	1970
Teamwork	1	10
Problem Solving	2	12
Interpersonal Skills	3	13
Oral Communication	4	4
Listening Skills	5	5
Personal Career Development	6	6
Creative Thinking	7	7
Leadership	8	8
Goal Setting/Motivation	9	9
Writing	10	1
Organizational Effectiveness	11	11
Computational Skills	12	2
Reading Skills	13	3

2

## Mugs and doughnuts

Geometry

- ✓ Distances, angles, curvature.
- ✓ “Metallic mathematics”.

Topology

- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.

2

## Mugs and doughnuts

### Geometry

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### Topology

- ✓ Overall shape, deformations.
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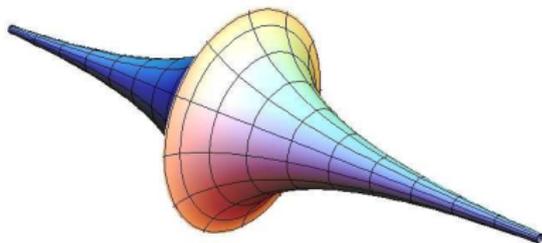
## Mugs and doughnuts

### Geometry

- ✓ Distances, angles, curvature.
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- ✓ Examples of applications:
  - 1) surveying;
  - 2) construction;
  - 3) astronomy;
  - 4) the shape of the universe.

### Topology

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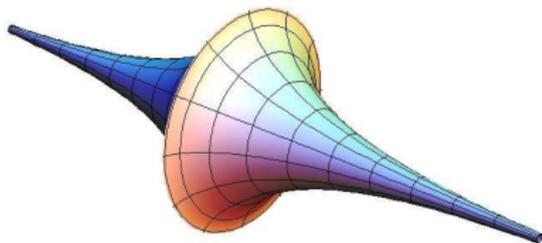


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## Mugs and doughnuts

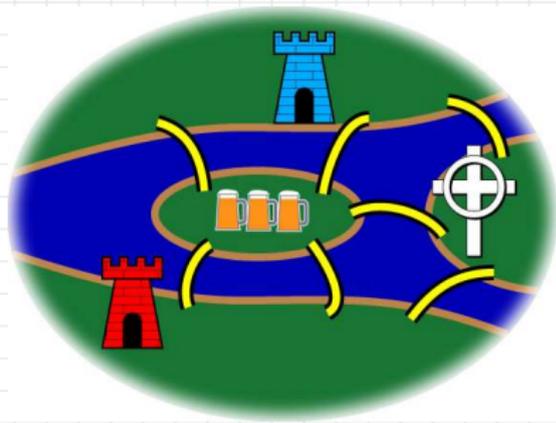
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### Topology

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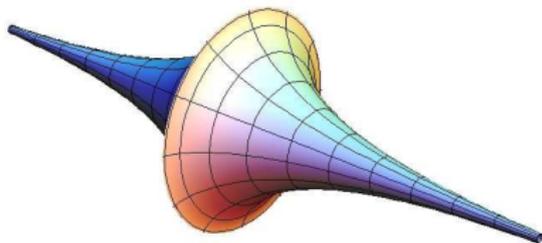


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## Mugs and doughnuts

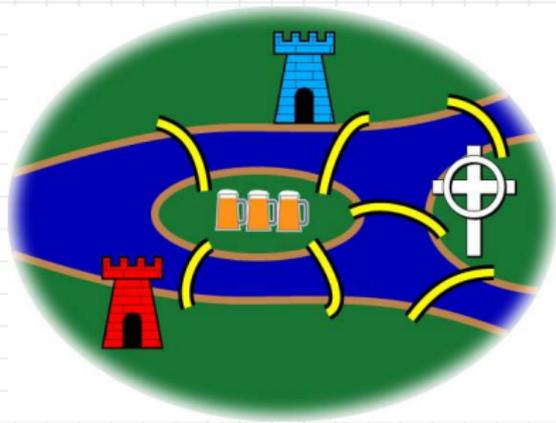
### Geometry

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### Topology

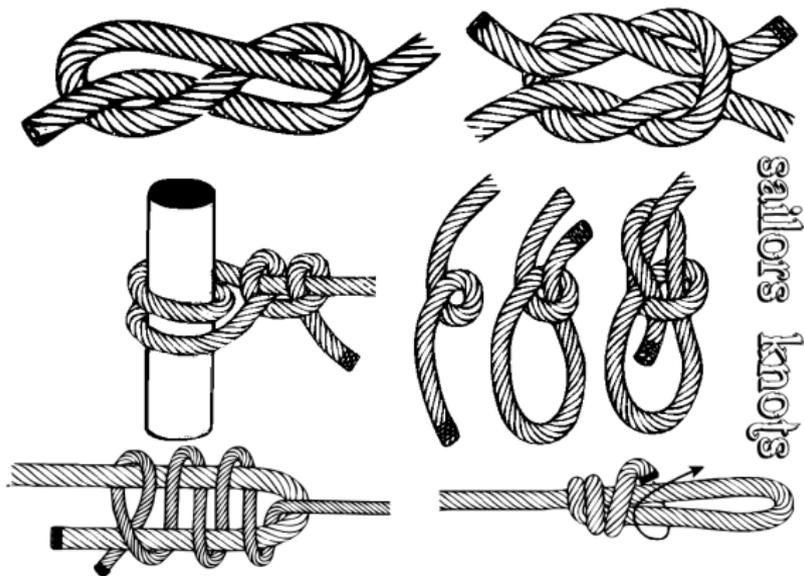
- ✓ Overall shape, deformations.
- ✓ “Clay mathematics”.
- ✓ mug = doughnut
- ✓ Examples of applications:
  - 1) Seven Bridges of Königsberg Problem;
  - 2) knot theory.



## Knots and braids

**Knots and braids surround us:**

- ✓ rope knots in sailing, mountaineering



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## Knots and braids

✓ ties



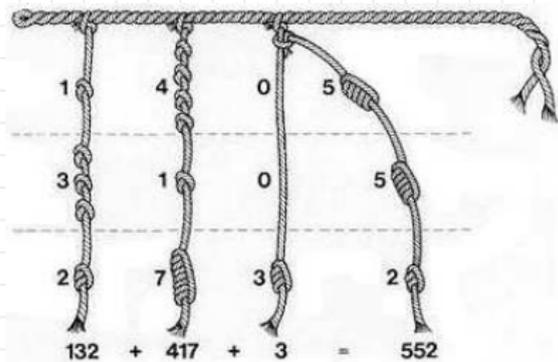
✓ hairstyles

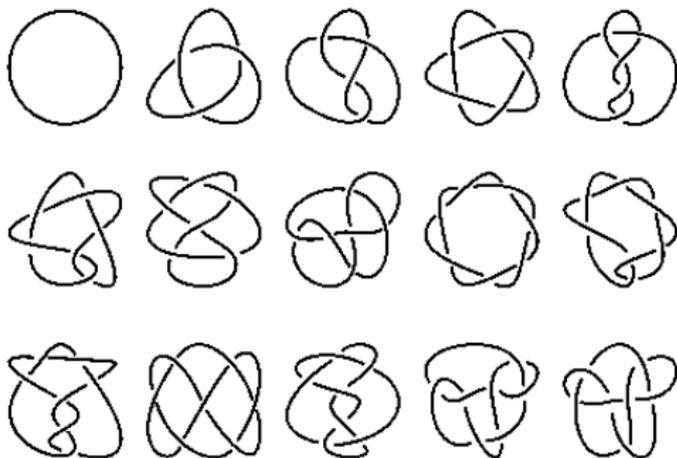


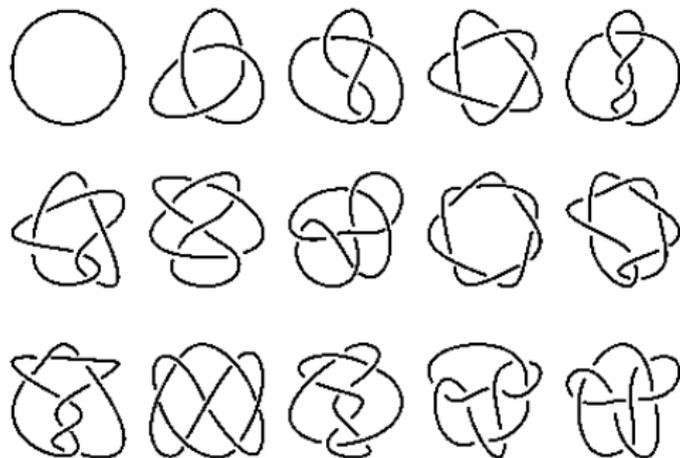
✓ decoration, religion



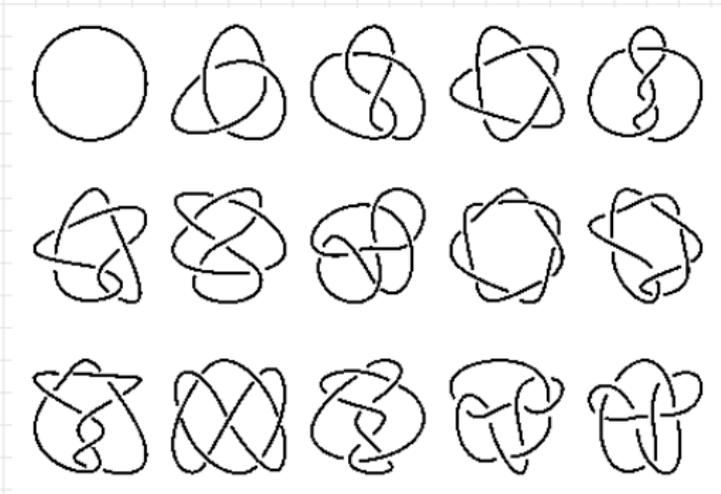
✓ quipu: used by Incas  
for recording information







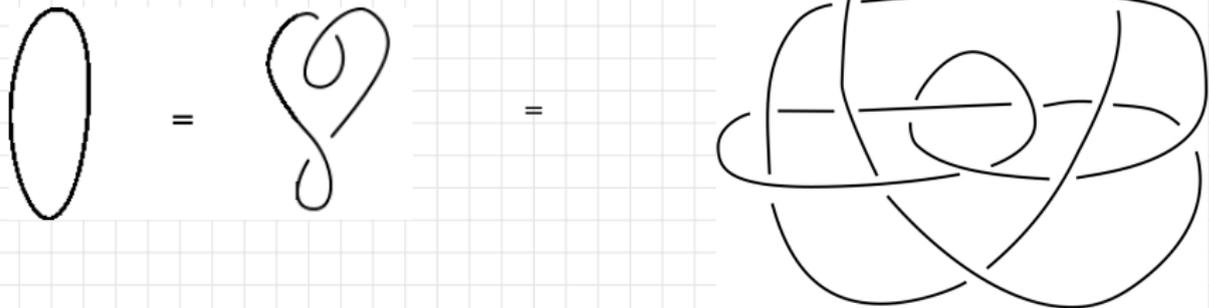
<b>nature</b>	many objects: different material, size, usage
<b>mathematics</b>	one abstraction: shape



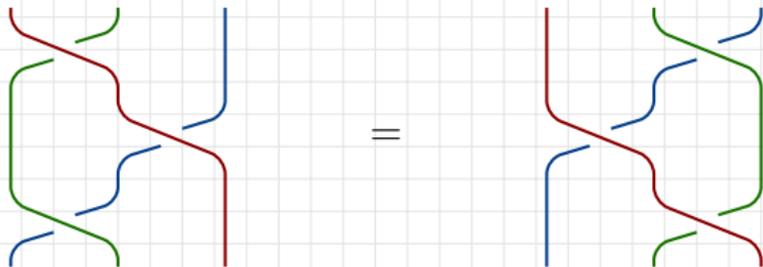
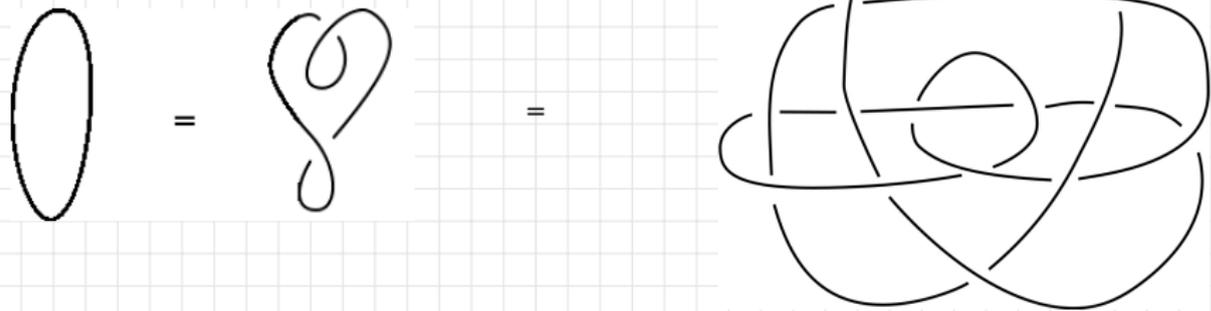
<b>nature</b>	many objects: different material, size, usage
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**Mathematics** begins when many objects are replaced with one abstraction.

In mathematics, knots and braids are considered up to deformation:

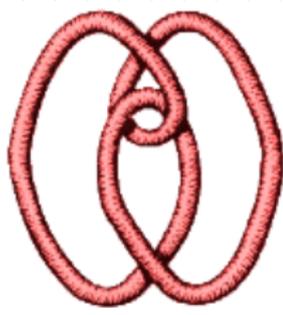
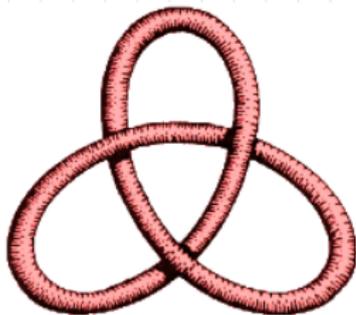
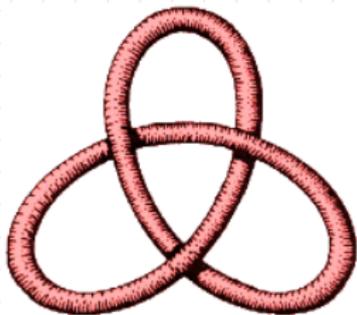


In mathematics, knots and braids are considered up to deformation:



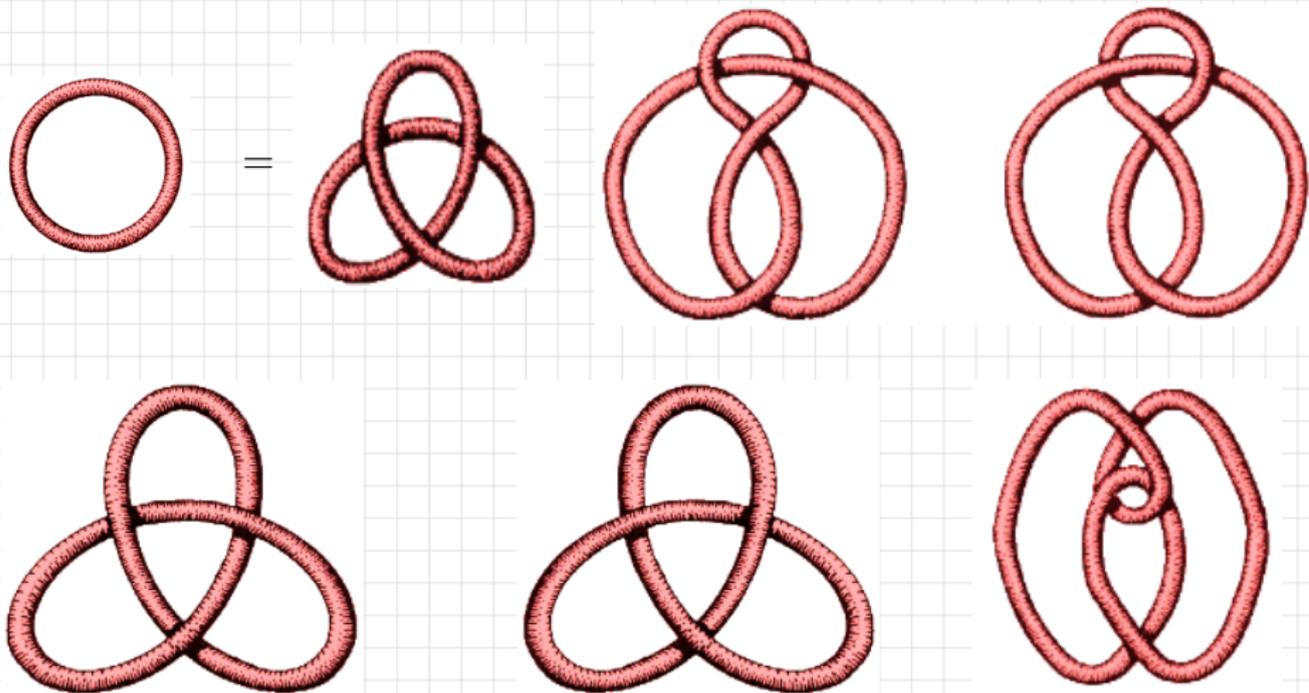
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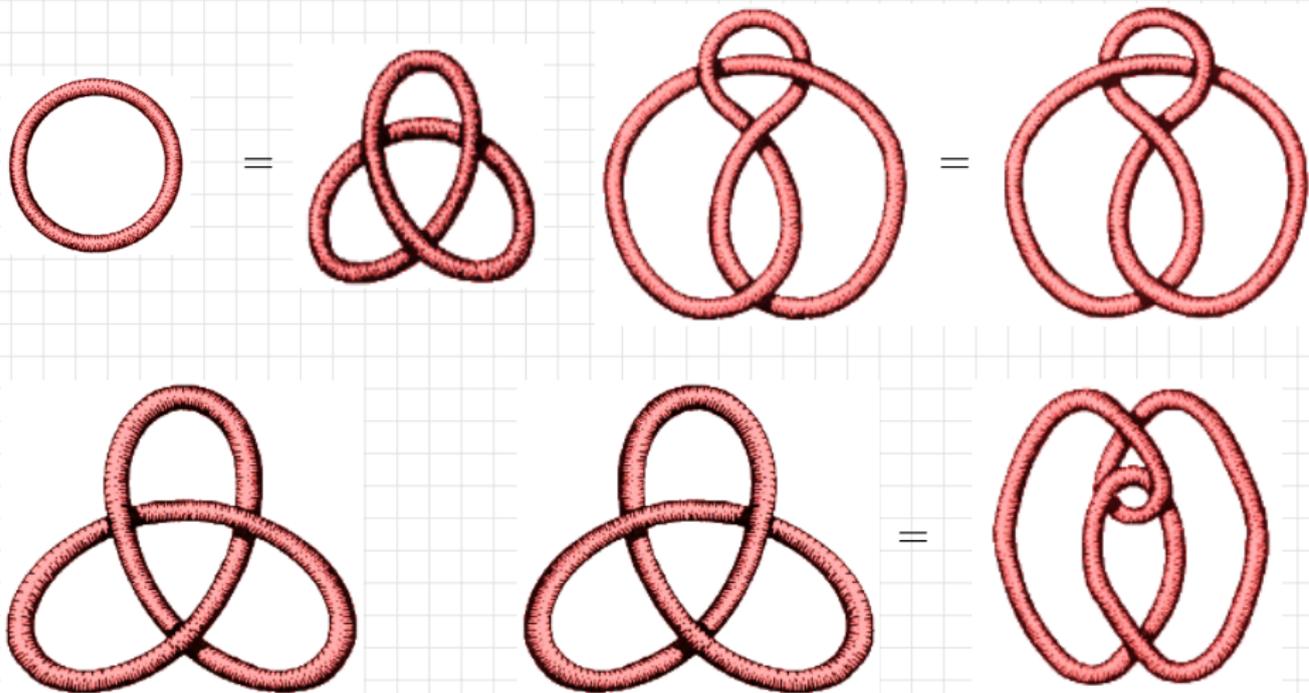
## How to distinguish knots?



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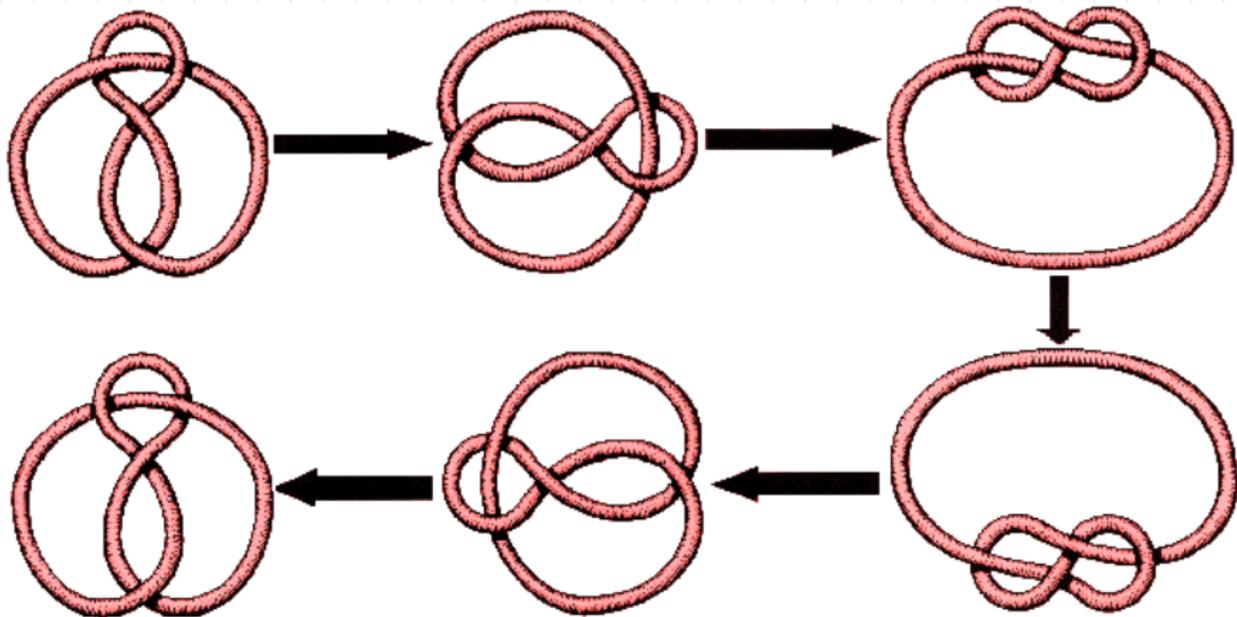
# How to distinguish knots?





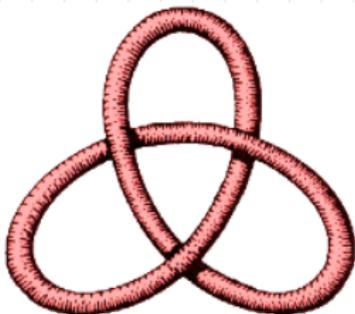
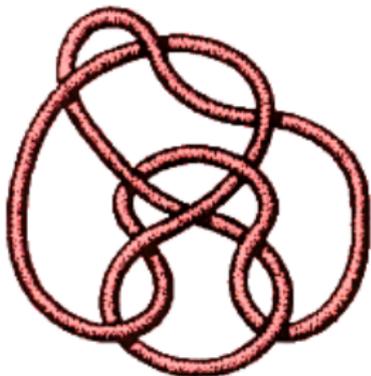
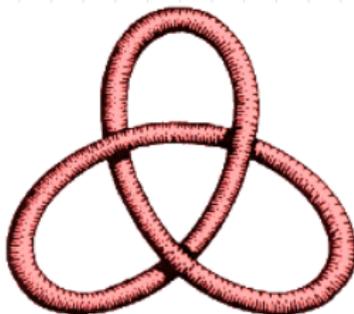
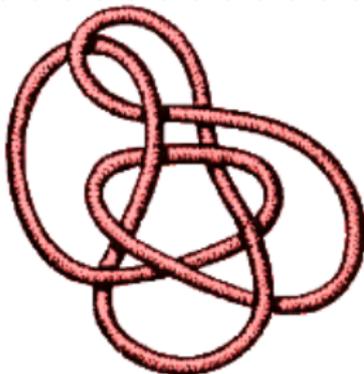
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## How to distinguish knots?



# How to distinguish knots?

It is a difficult problem:

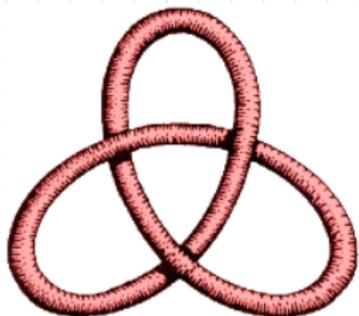
 $\neq$  $=$ 

6

# Knot invariants



?  
 $\neq$

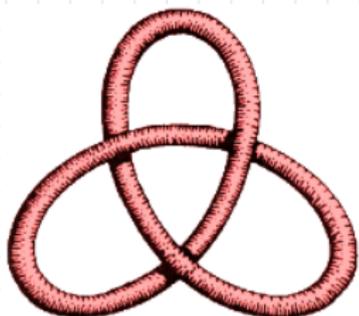
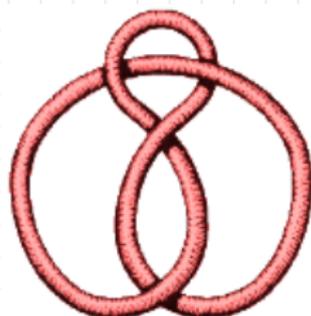


?  
 $\neq$



6

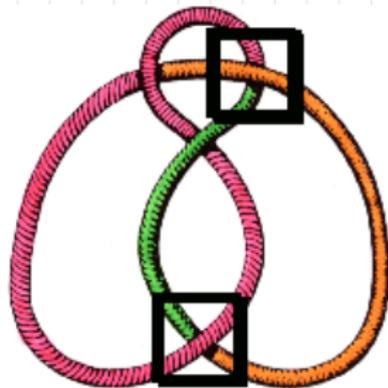
## Knot invariants

 $?$   
 $\neq$  $?$   
 $\neq$ 

**Knot colourings: 3 colours**



**Rule:** If a knot  $K'$  is a deformation of a knot  $K$ , and  $K$  can be coloured by 3 colours, then  $K'$  can be coloured by 3 colours as well.

 $\neq$  $\neq$ 

no

**Can be coloured ?**

yes

no



## Knot theory: history

1867: Peter Tait experimented with **smoke rings**;





## Knot theory: history

1867: Peter Tait experimented with **smoke rings**;



Lord Kelvin (Thomson): atoms = knotted tubes of ether



**C (Carbon)**



**O (Oxygen)**



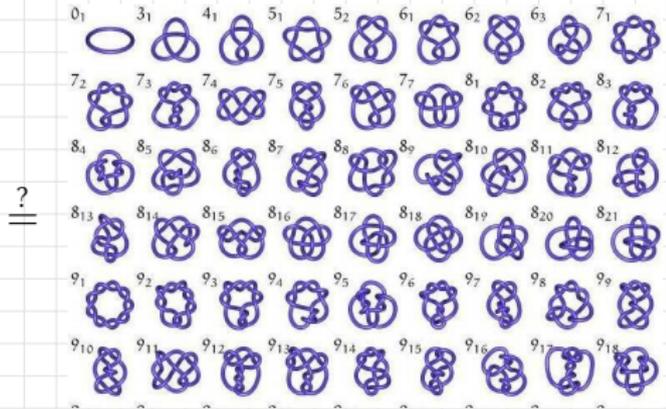
**H (Hydrogen)**

# 7 Knot theory: history

**PERIODIC TABLE OF THE ELEMENTS**

● Alkali metals    ● Metalloids    ● Lanthanides  
● Alkaline earth metals    ● Nonmetals    ● Actinides  
● Transition metals    ● Halogens  
● Post transition metals    ● Noble gases

1 H (1.0079)																	2 He (4.0026)
3 Li (6.941)	4 Be (9.0122)											5 B (10.811)	6 C (12.011)	7 N (14.007)	8 O (15.999)	9 F (18.998)	10 Ne (20.180)
11 Na (22.990)	12 Mg (24.305)											13 Al (26.982)	14 Si (28.086)	15 P (30.974)	16 S (32.06)	17 Cl (35.453)	18 Ar (39.948)
19 K (39.098)	20 Ca (40.078)	21 Sc (44.956)	22 Ti (47.88)	23 V (50.942)	24 Cr (51.996)	25 Mn (54.938)	26 Fe (55.845)	27 Co (58.933)	28 Ni (58.693)	29 Cu (63.546)	30 Zn (65.38)	31 Ga (69.723)	32 Ge (72.64)	33 As (74.922)	34 Se (78.96)	35 Br (79.904)	36 Kr (83.80)
37 Rb (85.468)	38 Sr (87.62)	39 Y (88.906)	40 Zr (91.224)	41 Nb (92.906)	42 Mo (95.94)	43 Tc (98.906)	44 Ru (101.07)	45 Rh (101.07)	46 Pd (106.36)	47 Ag (107.868)	48 Cd (112.41)	49 In (114.82)	50 Sn (118.71)	51 Sb (121.76)	52 Te (127.6)	53 I (126.905)	54 Xe (131.29)
55 Cs (132.91)	56 Ba (137.33)	57-71 La-Lu (138.905)	72 Hf (178.49)	73 Ta (180.948)	74 W (183.84)	75 Re (186.207)	76 Os (190.23)	77 Ir (192.22)	78 Pt (195.084)	79 Au (196.967)	80 Hg (200.59)	81 Tl (204.38)	82 Pb (207.2)	83 Bi (208.98)	84 Po ([209])	85 At ([210])	86 Rn ([222])
87 Fr ([223])	88 Ra ([226])	89-103 Ac-Lr ([227])	104 Rf ([261])	105 Db ([262])	106 Sg ([263])	107 Bh ([264])	108 Hs ([265])	109 Mt ([266])	110 Uun ([267])	111 Uuq ([268])	112 Uub ([269])	113 Uuq ([270])	114 Uuq ([271])	115 Uuq ([272])	116 Uuq ([273])	117 Uuq ([274])	118 Uuq ([276])
89 La (138.91)	90 Ce (140.12)	91 Pr (140.91)	92 Nd (144.24)	93 Pm ([145])	94 Sm (150.36)	95 Eu (151.96)	96 Gd (157.25)	97 Tb (158.93)	98 Dy (162.50)	99 Ho (164.93)	100 Er (167.26)	101 Tm (168.93)	102 Yb (173.04)	103 Lu (174.97)			
105 Ac ([227])	106 Th (232.04)	107 Pa (231.04)	108 U (238.03)	109 Np (237)	110 Pu (244)	111 Am (243)	112 Cm (247)	113 Bk (247)	114 Cf (251)	115 Es (252)	116 Fm (257)	117 Md (258)	118 No (259)	119 Lr (262)			



# 7 Knot theory: history

**PERIODIC TABLE OF THE ELEMENTS**

Legend:

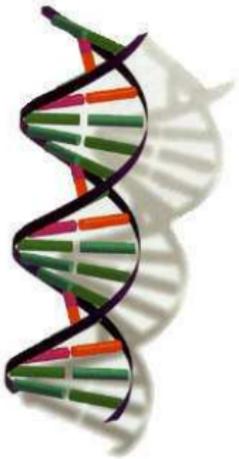
- Alkali metals (Yellow)
- Alkaline earth metals (Orange)
- Transition metals (Red)
- Post transition metals (Purple)
- Metalloids (Green)
- Nonmetals (Blue)
- Halogens (Light Blue)
- Noble gases (Light Green)
- Lanthanides (Pink)
- Actinides (Light Purple)

1 H 1.00794	2 He 4.002602																	10 Ne 20.1797	11 Na 22.98976928	12 Mg 24.304											18 Ar 39.9481634	19 K 39.0983	20 Ca 40.078											36 Kr 83.798	37 Rb 85.4678	38 Sr 87.62											54 Xe 131.29	55 Cs 132.90545196	56 Ba 137.327											86 Rn 222	87 Fr [223]	88 Ra [226]																																																																																											
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## Knot theory: applications

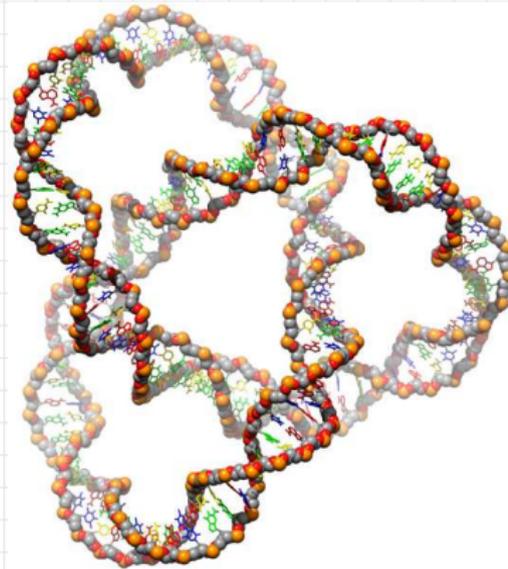
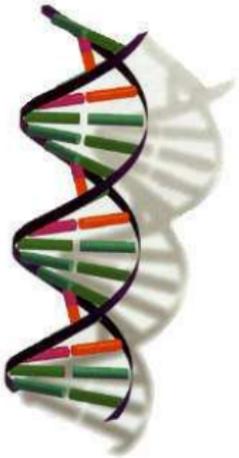
✓ **Biology:** DNA molecules.



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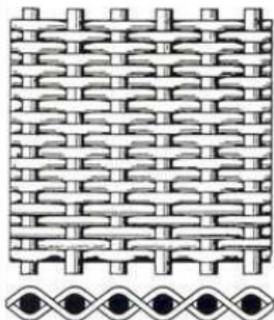
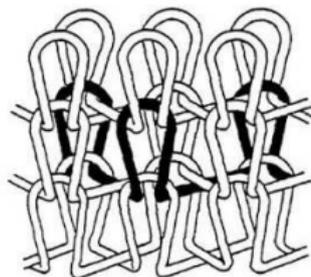
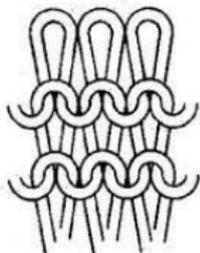
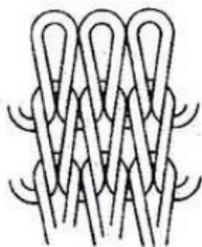
## Knot theory: applications

✓ **Biology:** DNA molecules.

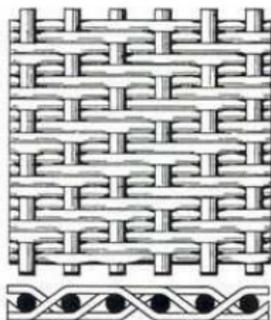


Some enzymes can cut, twist, and reconnect the DNA.  
This changes the properties of the DNA.  
Knot theory is used to detect the action of enzymes.

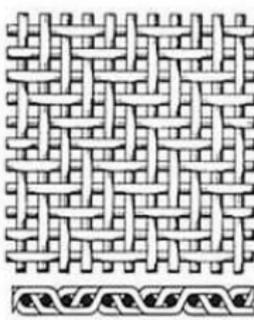
✓ **Textile:** weaving patterns.



PLAIN DUTCH WEAVE

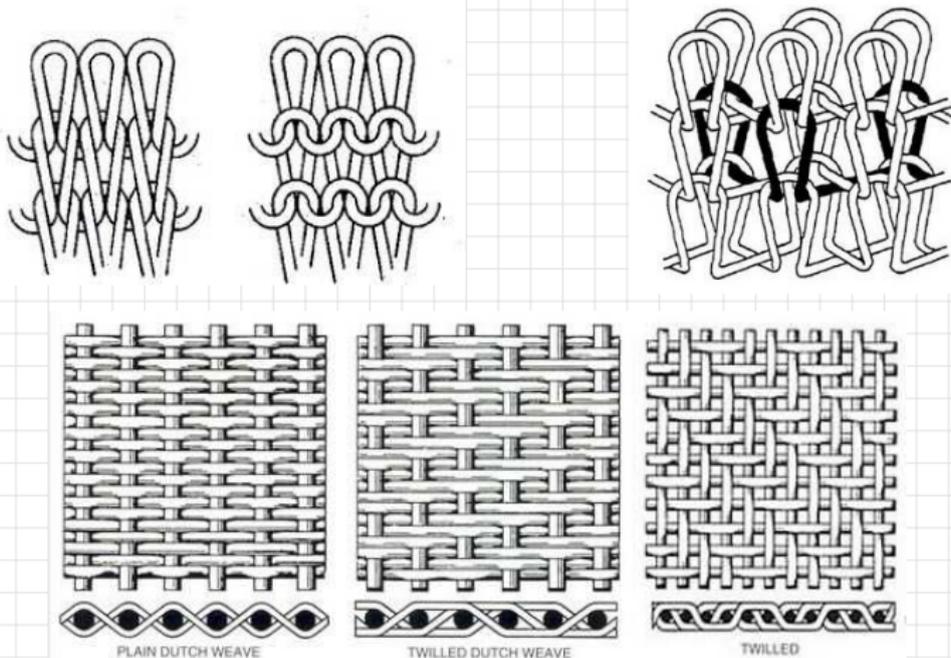


TWILLED DUTCH WEAVE



TWILLED

✓ **Textile:** weaving patterns.



✓ **The shape of the universe** question: all 3-dimensional spaces can be encoded by knots.